▸ TEODOR J. STĘPIEŃ, ŁUKASZ T. STĘPIEŃ, *On the consistency of Peano's Arithmetic System.*
Pedagogical University, ul. Podchorazych 2, 30-084 Kraków, Poland.
E-mail: tstepien@ap.krakow.pl.
Pedagogical University, ul. Podchorazych 2, 30-084 Kraków, Poland.
E-mail: lstepien@ap.krakow.pl, stepien50@poczta.onet.pl.

§1. Terminology. Let $\rightarrow, \sim, \wedge, \equiv$ denote connectives of implication , negation, conjuction and equivalence, respectively . Next Sp denotes the set of all well-formed formulas of Peano's Arithmetic System. R_{Sp} denotes the set of all rules over Sp. For any $X \subseteq Sp$ and for any $R \subseteq R_{Sp}, Cn(R, X)$ is the smallest subset of Sp containing X and closed under the rules of R . The couple $\langle R, X\rangle$ is called a system, whenever $R \subseteq R_{Sp}$ and $X \subseteq Sp$. Next r_o denotes Modus Ponens, r_+ denotes the generalization rule and $R_{o+} = \{r_o, r_+\}$ (see [1]). We use $\Rightarrow, \neg, \&, \Leftrightarrow, \forall, \exists$ as metalogical symbols . L_2 and A_r denote the set of all logical axioms and the set of all specific axioms in Peano's Arithmetic System, respectively (see [2]). Hence $\langle R_{o+}, L_2 \cup A_r\rangle$ is Peano's Arithmetic System.

The traditional consistency:

Definition. $\langle R, X\rangle \in Cns^T \Leftrightarrow (\neg\exists\alpha \in Sp)[\alpha \in Cn(R, X) \ \ \& \sim \alpha \in Cn(R, X)]$

§2. THE MAIN RESULT. THEOREM. $\langle R_{0+}, L_2 \cup A_r\rangle \in Cns^T$.
Proof. (elementary and combinatorial, see [3]).

§3. References.

[1] W. Pogorzelski, ***The classical calculus of quantifiers***, PWN, Warszawa 1981 (in Polish).

[2] H. Rasiowa, ***Introduction to contemporary mathematics***, PWN, Warszawa 1977 (in Polish).

[3] J. von Neumann, *Die formalistische Grundlegung der Mathematik*, ***Erkenntnis***, vol. 2 (1931),pp. 116–121.