

- TODOR TSANKOV, *The additive group of the rationals is not automatic.*  
Équipe d'Analyse Fonctionnelle, Université Paris 6, 4 Place Jussieu, 75252 Paris,  
France.  
*E-mail:* [todor@math.jussieu.fr](mailto:todor@math.jussieu.fr).  
*URL Address:* <http://people.math.jussieu.fr/~todor/>.

A countable relational structure is called *automatic* if there exists a coding of the elements of its domain by strings in a finite alphabet such that both the domain and the relations are regular languages (recognized by finite automata). One can transfer the definition to languages with function symbols by considering the graphs of the functions. The prototypical example of an automatic structure is the group of integers  $(\mathbf{Z}, +)$ , where each integer is coded by its decimal representation and addition is done by the normal carry procedure. Structures that admit an automatic presentation have pleasant properties from a computational perspective: their first order theories and model checking problems are decidable in a simple manner.

The main result that I am going to present is that the additive group of the rationals does not have such a presentation. This answers a question of Khoussainov. The proof also applies to certain other abelian groups, for example, torsion-free groups that are  $p$ -divisible for infinitely many primes  $p$ , or some torsion groups, like  $\mathbf{Q}/\mathbf{Z}$ . The proof is combinatorial and relies most notably on Freiman's structure theorem for sets with a small doubling constant.