▶ ALAN R. WOODS, *On the probability of absolute truth for And/Or Boolean formulas*.
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An *And/Or formula* such as $((X_1 \vee \overline{X}_2) \wedge X_3) \vee (\overline{X}_1 \wedge X_3)$ is a Boolean formula formed from literals using binary $\wedge$ and $\vee$ connectives (and brackets). Its *size* m is the number of occurrences of literals. ($m = 5$ in the example.) Suppose the variables are drawn from among $X_1, \ldots, X_n$. Let T_m denote the total number of And/Or formulas of size m in these n variables, and $T_m(True)$ be the number of these which are *tautologies*. A natural definition of the probability of a tautology is

$$P_n(True) = \lim_{m \to \infty} \frac{T_m(True)}{T_m} \; .$$

A second natural notion of probability is defined by generating a formula by means of a *Galton–Watson random branching process*. Throw a fair coin. If it is *heads*, throw a fair $2n$-sided die to choose a literal and then stop. If it is *tails*, throw the coin again to choose $\wedge$ or $\vee$ as the principal connective; then repeat the process to construct the left and right subformulas. Let $\pi_n(True)$ be the probability that the formula generated is a tautology.

THEOREM 1. (With Danièle Gardy [1].) $\pi_n(True) < P_n(True)$ *for all* n.

THEOREM 2. $\pi_n(True) \sim \dfrac{1}{4\,n}$ *and* $P_n(True) \sim \dfrac{3}{4\,n}$ *as* $n \to \infty$.

The probability that a random formula defines other simple Boolean functions such as a literal (as in the example above) can also be analysed.

[1] DANIÈLE GARDY AND ALAN R. WOODS, *And/or tree probabilities of Boolean functions*, In: *2005 International Conference on Analysis of Algorithms* (Conrado Martínez editor), ***Discrete Mathematics and Theoretical Computer Science Proceedings***, vol. AD (2005) pp. 139–146.