

- MARAT KH. FAIZRAHMANOV, *Splitting and antisplitting theorems in classes of low degrees.*

Kazan State University, Kazan, Russia.

E-mail: Marat.Faizrahmanov@ksu.ru.

A Turing degree $\mathbf{a} = \text{deg}(A)$ is called *superlow* if $A' \leq_{tt} \emptyset'$. A Turing degree \mathbf{a} is called *totally ω -c.e.* if every function $g \leq_T a$ is ω -c.e. It is known that each superlow degree is totally ω -c.e. Downey, Greenberg and Weber (2007) have proved that a degree \mathbf{a} is totally ω -c.e. if and only if \mathbf{a} does not bound a critical triple. Let \mathbf{J} be the partially ordered set of least upper bounds of superlow c.e. degrees and let \mathbf{C} be the upper semilattice of all c.e. degrees.

In the the following theorem the class Δ_a^{-1} is the Δ -level of the Ershov Hierarchy corresponding to the ordinal notation $a \in O$.

Theorem 1. For all notations $a \in O$ there is a low 2-c.e. set D , such that for all 2-c.e. sets E and F , if $E \in \Delta_a^{-1}$ and $F \in \Delta_a^{-1}$, then $D \not\equiv E \oplus F$.

Corollary 2. The low c.e. degrees and the low 2-c.e. degrees are not elementary equivalent.

Theorem 3. For all superlow c.e. degrees $\mathbf{b}_0, \mathbf{b}_1, \mathbf{b}_2$ there are superlow c.e. degrees $\mathbf{a}_0, \mathbf{a}_1, \mathbf{a}_2$, such that $\mathbf{b}_0 \cup \mathbf{b}_1 \cup \mathbf{b}_2 = \mathbf{a}_0 \cup \mathbf{a}_1 = \mathbf{a}_0 \cup \mathbf{a}_2 = \mathbf{a}_1 \cup \mathbf{a}_2$.

Corollary 4. The \mathbf{J} is an upper semilattice.

Theorem 5. There is c.e. degree \mathbf{b} , such that for all c.e. degrees $\mathbf{a}_0, \mathbf{a}_1, \mathbf{a}_2$ if $\mathbf{b} = \mathbf{a}_0 \cup \mathbf{a}_1 = \mathbf{a}_0 \cup \mathbf{a}_2 = \mathbf{a}_1 \cup \mathbf{a}_2$ then exists some $i < 3$ such that \mathbf{a}_i is not totally ω -c.e.

Corollary 6. The upper semilattices \mathbf{C} and \mathbf{J} are not elementary equivalent.