

- NORMA B. GOETHE, NANCY BOYALLIAN, *Working tools and the special rigor of mathematical reasoning*.

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The modern conception of rigorous proof relies on a syntactic characterization of proof as sequence of sentences. Is proof only a matter of deductive logic, as the syntactic conception suggests? In the paper we argue, using material from Grosholz (2007), that this is not the case. In contrast to formal proofs, mathematical arguments serve many purposes: they establish definitions, formulate problems and explain solutions; they present procedures and display proofs formally and informally. Moreover, real proofs are not like formal syntactic proofs. Formalization in terms of predicate logic with its one-dimensional form of writing is only one form of representation among many. There is a reason mathematical results are typically written out differently. The different modes of presentation of arguments emerge from different traditions of inquiry that require polyvalent mathematical discourse. Grosholz shows that one and the same argument often requires the combination of different modes of representation: diagrams, proportions, equations, matrices, tables, schemata, natural language. Formal forms of reasoning must be surrounded by natural language that explains their use while the mathematicians acquired know-how which is what permits him to engage in problem-solving, remains largely implicit. For example, arguments may require that one and the same representation be used ambiguously to allow for the mathematician to present a novel perspective and for the reader to follow the reasoning. Including ambiguity in argument illustrates the interconnection between fruitfulness and rigorous argument with signs.

The paper will focus on this reasoning style that has been neglected by philosophers of mathematics: at home in the tradition of analysis, such style is at odds with the stringent requirements of rigor which underlie the syntactic conception of proof that requires making explicit everything essential to reasoning, so that in any complete proof of a theorem figures and other non syntactic forms of representation are dispensable.