▸ DAVID FERNÁNDEZ-DUQUE, *Dynamic topological completeness for the plane*.
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Dynamic Topological Logic ($\mathcal{DTL}$) is a tri-modal system for reasoning about topological dynamics, that is, about the action of a continuous function f on a topological space X. Here we will consider the fragment $\mathcal{DTL}^{\Box\bigcirc}$ (also called $\mathcal{S4C}$, as introduced by S. Artemov, J.M. Davoren and A. Nerode), which uses two modalities; $\Box$, interpreted as a topological interior operator, and $\bigcirc$, interpreted as the preimage under f.

Dynamic topological systems arise in many distinct branches of mathematics, where the exact structure of X and f may vary substantially; however, the first class of dynamical systems that springs to mind for most people is that where $X = \mathbb{R}^n$. As such, the problem of describing $\mathcal{DTL}^{\Box\bigcirc}$ on Euclidean space deserves special attention. More precisely, given a topological space X, we can define a logic $\mathcal{DTL}^{\Box\bigcirc}_X$ of all those formulas φ of $\mathcal{DTL}^{\Box\bigcirc}$ such that, whenever $f : X \to X$ is a continuous function and V is an interpretation of propositional variables by subsets of X,

$$\langle X, f\rangle \models \varphi.$$

The problem at hand is, then, to describe $\mathcal{DTL}^{\Box\bigcirc}_{\mathbb{R}^n}$.

The case $n = 1$ is a rather innocent-looking problem which is surprisingly challenging. It is a result of S. Slavnov, further developed by M. Nogin and A. Nogin that $\mathcal{DTL}^{\Box\bigcirc}_{\mathbb{R}}$ contains non-trivial valid formulas, although the complete logic is not yet fully understood. Slavnov had also noted that the $\Box, \bigcirc$ fragment is indeed complete for interpretations on $\{\mathbb{R}^n : n > 0\}$, but gave a construction which required n to be arbitrarily large.

Because of this it is a surprising fact that $\mathcal{S4C}$ is complete for $\mathbb{R}^2$. The proof uses an intriguing geometric construction; we will discuss some of the basic intuitions behind this construction and discuss how it exploits essential differences between the real line and the plane.