▸ THANASES PHEIDAS, KARIM ZAHIDI, *Elimination Theory for Addition and the Frobenius Map in Rings of Power Series.*
Department of Mathematics, University of Crete, P.O. BOX 4107, Heraklion, Crete, Greece.
E-mail: `pheidas@maths.uoc.gr`.
E-mail: `zahidi@logique.jussieu.fr`.

We present progress towards a positive answer to the following question:

QUESTION 1. *Let $\mathbf{F}_q$ be a finite field of characteristic p, with $q = p^n$ elements. Let t be a variable. Let $\mathbf{P}$ be a one-place function-symbol and $+$ a two-place function symbol. Consider the theory $T_{\mathbf{P}}$ of $\mathbf{F}_p[[t]]$ in the language $L_{\mathbf{P}}$ with non-logical symbols the elements of $\{+, \mathbf{P}, 0, 1, t\}$, the interpretation of $\mathbf{P}$ is the Frobenius map: $\mathbf{P}(x) = x^p$, $+$ represents usual addition and $0, 1, t$ the usual elements of $\mathbf{F}_q[[t]]$. Is the theory of $T_{\mathbf{P}}$ model-complete?*

We will discuss various interconnections with similar past results and current research goals. Some of these are the following.

(a) It is unknown whether the theory of $\mathbf{F}_q[[t]]$ as a ring (with the structure of addition and multiplication) is decidable. This contrasts the known result that the theory of $F[[t]]$ is decidable if F is a field of zero characteristic with a decidable theory (results of Kochen). Our results concern obviously a sub-theory of the ring-theory of $\mathbf{F}_q[[t]]$.

(b) We have showed in the past that the similar structure on a ring of polynomials $\mathbf{F}_q[t]$ is model-complete (hence decidable). This implies that in the structure of $\mathbf{F}_q[t]$ in the language which results by replacing $\mathbf{P}$ by a symbol $\mathbf{D}$ for differentiation ($\mathbf{D}(x) = \frac{dx}{dt}$), every formula is equivalent to an existential formula of $T_{\mathbf{P}}$. This provides an effective elimination theory for the structure of $\mathbf{F}_q[t]$ with addition and differentiation. If the answer to the Question above is positive, the similar statements hold for $\mathbf{F}_q[[t]]$. We are interested in knowing to what extent this elimination can be achieved uniformly in n and in p (i.e. as n and p vary). Our ultimate goal is to investigate the structure of addition and differentiation in ultra-products of rings $\mathbf{F}_q[t]$ and $\mathbf{F}_q[[t]]$. We intend to apply such knowledge to geometric questions that amount to relating the solutions of the reductions modulo p of a linear algebraic differential equation with coefficients in $\mathbf{Z}[t]$, as p varies.