▶ SEAN COX, *Consistency strength of nonregular ultrafilters.*
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A *nonregular ultrafilter* is a weak version of a countably complete ultrafilter which arose from classic questions in model theory about cardinalities of ultrapowers. In particular, if an ultrafilter U on ω_1 has the property that $|{}^{\omega_1}\omega/U| = \omega_1$, then U must be nonregular.

Nonregularity is a weakening of countable completeness. Although in ZFC there is never a countably complete ultrafilter over a cardinal like ω_n, it is consistent relative to large cardinals that there is a nonregular ultrafilter on ω_n ($n \geq 1$). For $n = 1$, such an ultrafilter can be obtained starting with ω many Woodin cardinals (see [4]). For $n = 2$, the known upper bounds are higher, in the realm of huge cardinals (see [3] and [2]).

The best-known lower bound for the consistency strength of a nonregular ultrafilter on ω_1 is a stationary limit of measurable cardinals, due to Deiser and Donder [1]. This was an improvement on previous work by Ketonen and Donder/Jensen/Koppelberg. I will discuss my extensions of this work, particularly for nonregular ultrafilters on ω_2.

[1] Deiser, Oliver; Donder, Dieter, *Canonical functions, non-regular ultrafilters and Ulam's problem on ω_1*, ***Journal of Symbolic Logic***, vol. 68 (2003), no. 3, pp. 713–739.

[2] Foreman, Matthew, *An $\aleph_1$-dense ideal on $\aleph_2$*, ***Israel Journal of Mathematics***, vol. 108 (1998), pp. 253–290.

[3] Foreman, M.; Magidor, M.; Shelah, S., *Martin's maximum, saturated ideals and nonregular ultrafilters. II*, ***Annals of Mathematics***, vol. 127 (1988), no. 3, pp. 521–545.

[4] Woodin, W. Hugh, *The axiom of determinacy, forcing axioms, and the nonstationary ideal*, ***de Gruyter Series in Logic and its Applications***, 1999.