▸ NIKOLAS VAPORIS, *The extension property up to n and T_n–projectivity.*
Department of Philosophy, University of Utrecht, Heidelberglaan 8, 3584 CS Utrecht, The Netherlands.
E-mail: `nick.vaporis@phil.uu.nl`.

We refine a well–known theorem of Ghilardi published in [1] which establishes the equivalence between the extension property and projectivity. The definitions of these two notions are as follows:

- A formula φ has the *extension property* if we can add a node below an arbitrary collection of rooted Kripke models of φ so that the obtained downwards extended model preserves the satisfaction of φ.
- A formula is *projective* if there exists a substitution σ such that $\vdash \sigma\varphi$ and $\varphi \vdash (\sigma p \leftrightarrow p)$ for every propositional letter p, where $\vdash$ denotes deduction in intuitionistic propositional logic (**IPC**).

In [3] Iemhoff refines the extension property so that the cardinality of the collection of models is not arbitrary but instead restricted by a natural number n. That definition gives naturally rise to the question of how can we refine the notion of projectivity in order to get a generalised version of Ghilardi's theorem. The answer involves the $\mathbf{T_n}$ logics, which are the intermediate logics of n–ary Kripke tree frames, e.g. $\mathbf{T_1}$ is the logic of linear frames, $\mathbf{T_2}$ is the logic of binary trees and so on (see paper [2] of Gabbay and de Jongh for the basic properties of these logics). As it turns out, in order to get an equivalent notion to the extension property up to n it suffices to change in the definition of projectivity the underlying logic from **IPC** to $\mathbf{T_n}$, i.e.

Definition 1. A formula is $\mathbf{T_n}$*–projective* if there exists a substitution σ such that $\vdash \sigma\varphi$ and $\varphi \vdash (\sigma p \leftrightarrow p)$ for every propositional letter p, where $\vdash$ denotes deduction in the $\mathbf{T_n}$–logic.

We can now state our main contribution which is the following theorem:

Theorem 2. *Given a unifiable formula φ and a natural number $n \geq 2$, it holds that φ has the extension property up to n if and only if it is T_n–projective.*

The proof of the theorem follows Ghilardi's proofline, therefore it is constructive in the sense that for every T_n–projective formula we get a substitution as a witness of projectivity.

[1] Silvio Ghilardi, *Unification in Intuitionistic Logic*, ***Journal of Symbolic Logic***, vol. 64 (1999), no. 2, pp. 859–880.

[2] Dov Gabbay, Dick de Jongh, *A Sequence of Decidable Finitely Axiomatizable Intermediate Logics with the Disjunction Property*, ***Journal of Symbolic Logic***, vol. 39 (1974), no. 1, pp. 67–78.

[3] Rosalie Iemhoff, *A(nother) Characterization of Intuitionistic Propositional Logic*, ***Annals of Pure and Applied Logic***, vol. 113 (2002), no. 1–3, pp. 161-173.