

- TAPANI HYTTINEN, VADIM KULIKOV, *Weak Ehrenfeucht-Fraïssé equivalences.*  
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A *Weak Ehrenfeucht-Fraïssé game on  $L$ -structures  $\mathcal{A}$  and  $\mathcal{B}$  of length  $\alpha$* , denoted by  $EF_\alpha^*(\mathcal{A}, \mathcal{B})$  or shorter, is played between two players I and II on two  $L$ -structures  $\mathcal{A}$  and  $\mathcal{B}$ , where  $L$  is a relational vocabulary. The players choose elements of the domains of the structures in  $\alpha$  moves, and in the end of the game the player II wins if the chosen structures are isomorphic. Otherwise player I wins.

The obvious difference of this to the ordinary Ehrenfeucht-Fraïssé game is that the isomorphism can be arbitrary whereas in the ordinary EF-game it should be determined by the moves of the players. In particular this game is not closed (in the sense of Gale-Stewart [3]). In our article we answer the following questions and in the talk we discuss some of them.

- Are the games  $EF_\omega$  and  $EF_\omega^*$  equivalent? This was solved already by Kueker in [1] in the context of cub-subsets of power sets. (Answer: yes)
- Are the games  $EF_\alpha$  and  $EF_\alpha^*$  equivalent for an ordinal  $\alpha$ ? (Answer: no)
- Are the games  $EF_\kappa$  and  $EF_\kappa^*$  always equivalent for a cardinal  $\kappa > \omega$ ? (Answer: for structures of size  $\kappa^{++}$  no, for  $\kappa = \omega_1$  and structures of size  $\aleph_2$ , independent of ZFC. Here we use results of [2])
- If structures are weakly  $\alpha$ -equivalent and  $\beta < \alpha$ , are they necessarily weakly  $\beta$ -equivalent? (Answer: no)
- Is  $EF_{\omega_1}^*$  necessarily determined? (Answer: independent of ZFC, if the size of the structures is  $\aleph_2$  and the answer is no, if the size of the structures is greater than  $\aleph_2$ ).

[1] D. W. Kueker *Countable approximations and Löwenheim-Skolem theorems*, , ***Annals of Math. Logic***, 11 (1977) 57-103.

[2] A. H. Mekler, S. Shelah and J. Väänänen: *The Ehrenfeucht-Fraïssé-game of length  $\omega_1$* . , ***Transactions of the American Mathematical Society***, 339:567-580, 1993., 11 (1977) 57-103.

[3] Gale, D. and Stewart, F. M. *Infinite games with perfect information*. In ***Contributions to the theory of games***, vol. 2, ***Annals of Mathematics Studies***, no. 28, pages 245–266. Princeton University Press, Princeton, N. J., 1953.