▸ KUANYSH ABESHEV, SERIKZHAN BADAEV, AND MANAT MUSTAFA, *Families without minimal numberings.*
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Computable numberings of the families of the sets from any given level of the arithmetical or the analytical hierarchy as well as the Ershov hierarchy are usually identified with the uniform sequences of sets from that level. More precisely, if $\{\mathcal{H}_n\}_{n\in\omega}$ is one of these hierarchies then a sequence A_0, A_1, $A_2, \dots$ of the subsets of ω is uniform in $\mathcal{H}_n$ if the set $\{\langle x, k\rangle : x \in A_k\}$ is in $\mathcal{H}_n$. Now, if we denote each set A_k of this sequence by $\alpha(k)$ then we get the computable numbering $\alpha : \omega \mapsto \mathcal{A}$. Here, the family $\mathcal{A}$ consists of all sets of that sequence. The set of all computable numberings of $\mathcal{A}$ is preordered by the reducibility relation of numberings: a numbering α is reducible to a numbering β if $\alpha = \beta \circ f$ for some computable function f. Minimal numberings of $\mathcal{A}$ are exactly those which are minimal in this preorder. It is easy to see that any finite family of the sets from each mentioned above hierarchy has a numbering which is reducible to any numbering of the family.

Theorems on an existence of the families without computable minimal numberings are the analogs of well known speed up theorems. The families of c.e. sets without computable minimal numberings were built in [1],[2]. In contrary, every infinite family of the sets from the levels above level one in the arithmetical hierarchy has infinitely many computable pairwise incomparable minimal numberings, [3]. We do not know whether there exists infinite families of the sets from any level of the analytical hierarchy which has no computable minimal numberings.

We prove that, for every finite level of the Ershov hierarchy, there exists a computable family of the sets from this level which has no any computable minimal numbering. And we propose conjecture that the same holds for the infinite levels of the Ershov hierarchy.

[1] V.V. V'yugin, *On some examples of upper semilattices of computable enumerations*, ***Algebra and Logic***, vol. 12 (1973), no. 5, pp. 287–296.

[2] S.A. Badaev, *On minimal enumerations*, ***Siberian Advances in Mathematics***, vol. 2 (1992), no. 1, pp. 1–30.

[3] S.A. Badaev and S.S. Goncharov, *On Rogers semilattices of families of arithmetical sets*, ***Algebra and Logic***, vol. 40 (2001), no. 5, pp. 283–291.