A quantified modal logic for rough sets.

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A structure of the form \((U, \{R_i\}_{i \in N})\), where \(N\) is an initial segment of \(\aleph\) representing \(N\) ‘sources’ and \(R_i\) is an equivalence relation on \(U\) representing the knowledge base of the \(i^{th}\) source, was considered in [3] to formally study the behavior of rough sets [4] in a multiple-source scenario. In this paper, we extend our study to consider structures of the form \((U, \{R_P\}_{P \subseteq N})\), called _multiple-source approximation systems for groups_ (denoted _MSAS\(^G\)_), where \(R_P\) represents the combined knowledge base of the finite group \(P\) of sources. \(R_P\) is an equivalence relation on \(U\) satisfying (i) \(R_P = \bigcap_{i \in P} R_i\), and (ii) \(R_\emptyset = U \times U\). A quantified propositional modal logic _LMSAS\(^G\)_, different from modal logic with propositional quantifiers [1] and modal predicate logic, is proposed with semantics based on _MSAS\(^G\)_s. The language has a set \(PV\) of propositional variables, and a set \(T\) of terms built with countable sets of constants and variables and a binary function symbol \(\star\). Formulae are got through the scheme: 

\[ p | \neg \alpha | \alpha \land \beta | A \alpha | [t] \alpha | t = s | \forall x \alpha, \]

where \(p \in PV\), \(t, s \in T\), and \(A\) is the global modal operator. Thus quantification ranges over modalities. The semantics is defined with the help of a function which maps a term \(t\) to a finite subset of \(N\), \(\star\) being translated as union of sets. The function determines which equivalence relation is to be used to evaluate a modality involving a term \(t\). A sound and complete axiomatization is given and some decidability problems are addressed. It is found that the modal systems \(B, S5\) and epistemic logics \(S5_{\text{D}}\) [2] are embedded in _LMSAS\(^G\)_\(5\). It is also observed that \(S5_{\text{D}}\) cannot replace _MSAS\(^G\)_ to serve our purpose. The semantics of \(S5_{\text{D}}\) considers a finite and fixed number of agents, thus giving a finite and fixed number of modalities in the language. But in the case of _LMSAS\(^G\)_\(5\), the number of sources is not fixed, and could also be countably infinite. So, unlike the case of epistemic logics, it is not possible here to refer to all/some sources using only the connectives \(\land, \lor\). The quantifiers \(\forall, \exists\) are used to achieve the task.


