▶ AGATHA WALCZAK-TYPKE, *Groups in non-elementary model theory and in set theory without the axiom of choice.*

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Two generalizations [1] of a seminal first-order result determine the possible properties of a group interpreted by the monster model in an excellent class or in the context of homogeneous model theory (HMT). The group in question is assumed to carry an ω-homogeneous pregeometry, and is obtained from the set of realizations of a so-called quasiminimal type; such a group is called a *quasi-minimal group*. This result is proved twice since the existing tools for HMT and for excellent classes are not compatible. The non-elementary result differs notably from the first-order original in that the possibility of a non-abelian quasi-minimal group is not ruled out. However, no non-abelian quasi-minimal group example is known, constituting a major open problem in model theory.

One can define various kinds of non-well-orderable sets which will exist only in universes of set theory without the axiom of choice. A set from such a class can serve as the domain of a structure. Previous research [2, 3] has demonstrated that structures on such sets can have interesting classification-theoretic properties.

I will discuss structures whose domains belong to one such class: quasi-amorphous sets. Previous research [4] has shown that quasi-amorphous groups must have properties very similar to those of quasi-minimal groups. Furthermore, while abelian quasi-amorphous groups are easy to construct, the existence of a non-abelian quasi-amorphous group is an open question. Previous research [3] has also shown that quasi-amorphous structures are characterizable by a complete sentence of an infinitary logic which only allows statement of countable size. This indicates that quasiamorphous groups may be best studied as part of a non-elementary model-theoretic context.

This leads to some open questions: Do quasi-amorphous structures fit "naturally" into any non-elementary context? Is it possible to construct a non-abelian quasi-amorphous group? Positive answers to these questions may provide a positive resolution to the open question concerning the existence of non-abelian quasi-minimal groups.

[1] Tapani Hyttinen, Olivier Lessmann, Saharon Shelah, *Interpreting groups and fields in some nonelementary classes*, ***Journal of Mathematical Logic***, vol. 5 (2005), no. 1, pp. 1–47.

[2] A. C. Walczak-Typke, *The first-order structure of weakly Dedekind-finite sets*, ***Journal of Symbolic Logic***, vol. 70 (2005), no. 4, pp. 1161–1170.

[3] ———, *A model-theoretic approach to structures in set theory without the axiom of choice*, ***Algebra, Logic, Set Theory: Festschrift für Ulrich Felgner zum 65. Geburtstag*** (Frieder Haug and Benedikt Löwe and Torsten Schatz, editors), College Publications, Kings College. London, 2007, pp. 255–289.

[4] John Truss, Phillip Creed, *On quasi-amorphous sets*, ***Archive for Mathematical Logic***, vol. 40 (2001), pp. 581–596.