▶ LARISA MAKSIMOVA, *Weak interpolation in extensions of Johansson's minimal logic*.
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A weak version of interpolation in extensions of Johansson's minimal logic is defined, and its equivalence to a weak version of Robinson's joint consistency is proved. We find some criteria for validity of WIP in extensions of the minimal logic.

Let L be a logic, $\vdash_L$ deducibility relation in L. *The weak interpolation property WIP* is defined as follows:

WIP. If $A, B \vdash_L \bot$, then there exists a formula C such that $A \vdash_L C$ and $B \vdash_L \neg C$, and all the variables of C are in both A and B.

Let L be any axiomatic extension of the minimal logic. An L-*theory* is a set T closed with respect to $\vdash_L$. An L-theory is *consistent* if it does not contain $\bot$. *The weak Robinson property WRP* is defined as follows:

WRP. Let T_1 and T_2 be two L-theories in the languages $\mathcal{L}_1$ and $\mathcal{L}_2$ respectively, $\mathcal{L}_0 = \mathcal{L}_1 \cap \mathcal{L}_2$, $T_{i0} = T_i \cap \mathcal{L}_0$. If the set $T_{10} \cup T_{20}$ in the common language is L-consistent, then $T_1 \cup T_2$ is L-consistent.

Theorem 1. *For any (predicate or propositional) extension L of the minimal logic, WIP is equivalent to WRP.*

The language of the minimal logic J contains $\&, \vee, \to, \bot$ as primitive; negation is defined by $\neg A = A \to \bot$. A formula is said to be *positive* if contains no occurrences of $\bot$. The logic J can be given by the calculus, which has the same axiom schemes as the positive intuitionistic calculus, and the only rule of inference is modus ponens. By a J-*logic* we mean an arbitrary set of formulas containing all the axioms of J and closed under modus ponens and substitution rules. We denote $\mathrm{Int} = \mathrm{J} + (\bot \to p)$, $\mathrm{Gl} = \mathrm{J} + (p \vee \neg p)$. A J-logic is *superintuitionistic* if it contains the intuitionistic logic Int, and *negative* if contains $\bot$.

Theorem 2. *For any* J-*logic L the following are equivalent: (1) L has WIP, (2) $L \cap L_1$ has WIP for any negative logic L_1, (3) $L \cap \mathrm{Neg}$ has WIP.*

Theorem 3. *Any propositional* J-*logic containing* $\mathrm{J} + \neg\neg(\bot \to p)$ *possesses WIP.*

The problem of weak interpolation is reducible to the same problem over Gl.

Theorem 4. *For any* J-*logic L, L has WIP if and only if $L + (p \vee \neg p)$ has WIP.*

Theorem 5. *There exists a* J-*logic, which contains* $\mathrm{Gl} = \mathrm{J} + (p \vee \neg p)$ *and does not possess the weak interpolation property.*

To prove that we consider two J-algebras **B** and **C**. The universe of **B** consists of four elements $\{a, b, \bot, \top\}$, where $a < b < \bot < \top$. The algebra **C** consists of five elements $\{c, d, e, \bot, \top\}$, where $e < x < \bot < \top$ for $x \in \{c, d\}$ and the elements c and d are incomparable. Let a J-logic L_1 be the set of all formulas valid in the both algebras **B** and **C**. Let

$$A(x, y) = (x \to y)\&((y \to x) \to x)\&(y \to \bot)\&((\bot \to y) \to y),$$

$$B(u, w) = ((u \to w) \to w)\&((w \to u) \to u)\&((u \vee w) \leftrightarrow \bot).$$

We prove that $A(x, y), B(u, w) \vdash_{L_1} \bot$, but this formula has no interpolant in L_1.