For any (predicate or propositional) extension of Johansson’s minimal logic is defined, and its equivalence to a weak version of Robinson’s joint consistency is proved. We find some criteria for validity of WIP in extensions of the minimal logic.

Let $L$ be a logic. If $A, B ⊢ ⊥$, then there exists a formula $C$ such that $A ⊢ C$ and $B ⊢ ¬C$, and all the variables of $C$ are in both $A$ and $B$.

Let $L$ be any axiomatic extension of the minimal logic. An $L$-theory is a set $T$ closed with respect to $⊢$. An $L$-theory is consistent if it does not contain $⊥$. The weak Robinson property WRP is defined as follows:

WRP. Let $T_1$ and $T_2$ be two $L$-theories in the languages $L_1$ and $L_2$ respectively, $L_0 = L_1 \cap L_2$, $T_{10} = T_1 \cap L_0$. If the set $T_{10} \cup T_{20}$ in the common language is $L$-consistent, then $T_1 \cup T_2$ is $L$-consistent.

**THEOREM 1.** For any (predicate or propositional) extension $L$ of the minimal logic, WIP is equivalent to WRP.

The language of the minimal logic $J$ contains $\&, \lor, \to, \bot$ as primitive; negation is defined by $¬A = A \to \bot$. A formula is said to be positive if contains no occurrences of $\bot$. The logic $J$ can be given by the calculus, which has the same axiom schemes as the positive intuitionistic calculus, and the only rule of inference is modus ponens. By a $J$-logic we mean an arbitrary set of formulas containing all the axioms of $J$ and closed under modus ponens and substitution rules. We denote $\text{Int} = J + (\bot \to p)$, $\text{Gl} = J + (p \lor ¬p)$. A $J$-logic is superintuitionistic if it contains the intuitionistic logic $\text{Int}$, and negative if contains $\bot$.

**THEOREM 2.** For any $J$-logic $L$ the following are equivalent: (1) $L$ has WIP, (2) $L \cap L_1$ has WIP for any negative logic $L_1$, (3) $L \cap \text{Neg}$ has WIP.

**THEOREM 3.** Any propositional $J$-logic containing $J + ¬(\bot \to p)$ possesses WIP.

The problem of weak interpolation is reducible to the same problem over Gl.

**THEOREM 4.** For any $J$-logic $L$, $L$ has WIP if and only if $L + (p \lor ¬p)$ has WIP.

**THEOREM 5.** There exists a $J$-logic, which contains $\text{Gl} = J + (p \lor ¬p)$ and does not possess the weak interpolation property.

To prove that we consider two $J$-algebras $B$ and $C$. The universe of $B$ consists of four elements $\{a, b, \bot, \top\}$, where $a < b < \bot < \top$. The algebra $C$ consists of five elements $\{c, d, e, \bot, \top\}$, where $e < x < \bot < \top$ for $x \in \{c, d\}$ and the elements $c$ and $d$ are incomparable. Let a $J$-logic $L_1$ be the set of all formulas valid in the both algebras $B$ and $C$. Let

$$A(x, y) = (x \to y) \& ((y \to x) \to x) \& (y \to \bot) \& ((\bot \to y) \to y),$$

$$B(u, w) = ((u \to w) \to w) \& ((w \to u) \to u) \& ((u \lor w) \leftrightarrow \bot).$$

We prove that $A(x, y), B(u, w) ⊢_{L_1} \bot$, but this formula has no interpolant in $L_1$. 

---

**LARISA MAKSIMOVA,** *Weak interpolation in extensions of Johansson’s minimal logic.* Sobolev Institute of Mathematics, Siberian Branch of Russian Acad. Sci., 630090 Novosibirsk, Russia.

E-mail: lmaksi@math.nsc.ru.