▸ ARTHUR W. APTER, STEPHEN C. JACKSON, BENEDIKT LÖWE, *Cofinality and measurability of the first three uncountable cardinals.*
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In ZFC, small cardinals such as $\aleph_1$, $\aleph_2$, and $\aleph_3$ cannot be measurable, as measurability implies strong inaccessibility; they cannot be singular either, as successor cardinals are always regular. But both of the mentioned results use the Axiom of Choice: in the Feferman-Lévy model, $\aleph_1$ has countable cofinality, in Jech's model, $\aleph_1$ is measurable, and in models of AD, both $\aleph_1$ and $\aleph_2$ are measurable and $\mathrm{cf}(\aleph_3) = \aleph_2$. Is it possible to control these properties simultaneously for the three cardinals $\aleph_1$, $\aleph_2$ and $\aleph_3$?

In this talk, we investigate all possible patterns of measurability and cofinality for the three mentioned cardinals. Combinatorially, there are exactly 60 ($= 3 \times 4 \times 5$) such patterns, of which 13 are impossible for trivial reasons. We reduce the remaining 47 patterns to eight *base cases* that we prove to be consistent relative to large cardinals. The consistency proofs heavily rely on the existence (assuming AD) of a cardinal $\kappa$ such that the triple $(\kappa, \kappa^+, \kappa^{++})$ satisfies the strong polarized partition property. This is a generalization of an unpublished theorem of Kechris from the 1980s.