▶ NURLAN KOGABAEV, *Undecidability of the theory of projective planes.*
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Shirshov suggested to treat projective planes as partial algebraic systems [1]. In the framework of this approach a *projective plane* is a structure $\langle A, (A^0, {}^0A), \cdot\rangle$ with a disjunction of A into two subsets $A^0 \cup {}^0A = A$, $A^0 \cap {}^0A = \emptyset$ and commutative partial operation "$\cdot$" which satisfy the following properties:

(1) $a{\cdot}b$ is defined iff $a{\neq}b$ and $a, b \in A^0$ (or $a, b \in {}^0A$) with the product $a{\cdot}b \in {}^0A$ ($a{\cdot}b \in A^0$ respectively);
(2) for all $a, b, c \in A$ if $a{\cdot}b$, $a{\cdot}c$, $(a{\cdot}b){\cdot}(a{\cdot}c)$ are defined, then $(a{\cdot}b){\cdot}(a{\cdot}c) = a$;
(3) there exist distinct $a, b, c, d \in A$ such that products $a{\cdot}b$, $b{\cdot}c$, $c{\cdot}d$, $d{\cdot}a$ are defined and pairwise distinct.

In any projective plane $\mathcal{A}$ we replace the partial operation by its graph and consider $\mathcal{A}$ as a predicate model. This allows us to apply methods of model theory and investigate the question of decidability of elementary theories [2].

In the present paper we prove that the class of symmetric, irreflexive graphs is relatively elementarily definable in the class of projective planes. Since the theory of symmetric, irreflexive graphs is hereditarily undecidable, we obtain the following results:

(1) *The class of all projective planes has hereditarily undecidable theory.*
(2) *The class of freely generated projective planes has hereditarily undecidable theory.*

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[1] A.I. Shirshov, A.A. Nikitin, *On the theory of projective planes*, ***Algebra and Logic***, vol. 20 (1981), no. 3, pp. 330–356.

[2] Yu.L. Ershov, I.A. Lavrov, A.D. Taimanov, M.A. Taitslin, *Elementary theories*, ***Russian Mathematical Surveys***, vol. 20 (1965), no. 4, pp. 35–105.