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E-mail: hakob_nalbandyan@yahoo.com. We compare the proof complexities in Frege systems with different modifications of the substitution rule.

We use the generally accepted concept of Frege system, the well-known notions of proof complexities (size and lines) and the notion of polynomial equivalence (by size and by lines) of the proof systems.

Let $\mathcal{F}$ be some Frege system. The substitution Frege system $S\mathcal{F}$ consists of Frege system $\mathcal{F}$ augmented with the substitution rule with inferences of the form $\frac{A}{A\sigma}$ for any substitution $\sigma = \begin{pmatrix} \varphi_{i_1} & \varphi_{i_2} & \dots & \varphi_{i_m} \\ p_{i_1} & p_{i_2} & \dots & p_{i_m} \end{pmatrix}$, where p_{i_j} $(1 \leqslant j \leqslant m)$ are the propositional variables, φ_{i_j} $(1 \leqslant j \leqslant m)$ are the propositional formulas, and $A\sigma$ denotes the result of applying of the substitution σ to formula A. Such substitution rule allows to use the simultaneous substitution of multiple formulas for multiple variables of A without any restrictions. If the depths of formulas φ_{i_j} $(1 \leqslant j \leqslant m)$ are restricted by some fixed d, then we have d- restricted substitution rule and we denote the corresponding system by $S^d\mathcal{F}$.

We prove that

1) given arbitrary $d_1 \geqslant 1$ and $d_2 \geqslant 1$, the systems $S^{d_1}\mathcal{F}$ and $S^{d_2}\mathcal{F}$ are polynomially equivalent (both by size and by lines),

2) given arbitrary d, the systems $S^d\mathcal{F}$ and $S\mathcal{F}$ are polynomially equivalent by size,

3) given arbitrary d, the minimal number of lines in a proof of tautology in $S^d\mathcal{F}$ can be exponentially larger than in $S\mathcal{F}$.

The analogous results have been obtained by first two authors for k-bounded substitution rule, which for some fixed k allows substitution for any no more than k variables at a time.

The main difference between these two weak substitution rules is the following:
for every $k \geqslant 1$ Frege system with k-bounded substitution rule has exponential speed-up by lines over the Frege system, but for every $d \geqslant 1$ $S^d\mathcal{F}$ and $\mathcal{F}$ are polynomially equivalent by lines.