SOME PROPERTIES OF COMPUTABLE NUMBERINGS IN VARIOUS LEVELS OF DIFFERENCE HIERARCHY

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$n$-computable enumerable sets

**Definition**

We call a set $A \subseteq \omega$ is $n$-computable enumerable if there are uniformly computable sequence of sets $\{A_s\}_{s \in \omega}$ such for all $x$,

$$x \notin A_0$$

$$A(x) = \lim_{s} A_s(x)$$

$$|\{s \in \omega | A_{s+1} \neq A_s\}| \leq n$$
Ershov’s hierarchy

Let $S$ – univalent notation system for constructive ordinals, $A \subseteq \omega$ and $\alpha$ – ordinal, which has notation $a$ in $S$.

**Definition**

Set $A \subseteq \omega$ in level $\Sigma_{\alpha}^{-1}$ of Ershov’s hierarchy (or $A$ is $\Sigma_{\alpha}^{-1}$-set), if there exist partially computable function $\Psi$, and for all $x$,

$$x \in A \rightarrow \exists \lambda (\Psi(\lambda, x) \downarrow \text{ and } A(x) = \Psi((\mu \lambda < \alpha)_S(\Psi((\lambda)_S, x) \downarrow, x))$$

$$x \not\in A \rightarrow \text{ or } \forall \lambda (\Psi(\lambda, x) \uparrow), \text{ or } \exists \lambda (\Psi(\lambda, x) \downarrow \text{ and } A(x) = \Psi((\mu \lambda < \alpha)_S(\Psi((\lambda)_S, x) \downarrow, x))$$.
Some definitions

$\Delta_{\alpha}^{-1}$-sets
finite levels of difference hierarchy

The End

Ershov’s hierarchy

Definition

A in level $\Pi_{\alpha}^{-1}$ of Ershov’s hierarchy, if $\overline{A} \in \Sigma_{\alpha}^{-1}$

A in level $\Delta_{\alpha}^{-1}$ of Ershov’s hierarchy, if $A$ and $\overline{A}$ are $\Sigma_{\alpha}^{-1}$-sets, in other words $\Delta_{\alpha}^{-1} = \Sigma_{\alpha}^{-1} \cap \Pi_{\alpha}^{-1}$. 
Definition

Numbering of family $S$ is a map $\nu$ from $\omega$ onto the family $S$.

Definition

Numbering $\eta$ is called $\Sigma_{\alpha}^{-1}$-computable, if set $\{<x, y> | y \in \eta x\}$ is a $\Sigma_{\alpha}^{-1}$-set and $\Delta_{\alpha}^{-1}$-computable, if $\{<x, y> | y \in \eta x\}$ in level $\Delta_{\alpha}^{-1}$.
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Numberings

Definition

Numbering $\eta$ is called Friedberg numbering, if for all $n \neq m$
$\eta_n \neq \eta_m$.

Definition

Numbering $\mu$ is called minimal, if for all numberings $\nu_n$ from reducing $\nu$ to $\mu$ goes, that $\nu$ is equivalent to $\mu$. 
Preposition

There is no universal computable function for family of all computable sets.

Theorem

There is no $\Delta^{-1}_\alpha$-computable numbering for family of all $\Delta^{-1}_\alpha$-sets.
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Friedberg theorem

**Theorem**

There is effective enumeration of the family of all computable enumerable sets without repetition.

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(Goncharov, Lemp, Solomon) For all $n$ there is $\Sigma_{n}^{-1}$-computable Friedberg numbering for family of all $\Sigma_{n}^{-1}$-sets.
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(Goncharov, Lemp, Solomon) For all $n$ there is $\Sigma^{-1}_n$-computable Friedberg numbering for family of all $\Sigma^{-1}_n$-sets.
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Friedberg numberings

Theorem

For all \( n \) there is \( \Sigma_{2n}^{-1} \)-computable Friedberg numbering for family of all \( \Sigma_n^{-1} \)-sets. And there is computable function, which \( m \)-reduces Friedberg numbering for family of all \( \Sigma_{n-1}^{-1} \)-sets to Friedberg numbering for family of all \( \Sigma_n^{-1} \)-sets. Moreover, numbering and function are constructed uniformly on \( n \).
Theorem

Let $\beta^n$ is numbering, which is constructed in previous theorem. Define $\gamma$:

$$\gamma_n = \beta^{n_1}_{n_2},$$

where $n = < n_1, n_2 >$. $\gamma_n$ is $\Delta^-_1$-computable minimal numberings for family of all sets from $\bigcup_{k \in \omega} \Sigma^-_{k}$.
Thanks for your attention!:}