On the question of consistence of the semantic $\mu$-prediction

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Sofia 2009
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• Knowledge of statistical character is captured by probability distributions, truth values are generalized to probabilistic.

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There are micro- (logical) and macro- (probabilistic) levels.

Current decisions are made in two-valued classical logic, so consistency of probabilistic theories/predictions (statistical ambiguity problem) is a very important question of AI.

Note that any $\phi$ should be examined both with its negation: each of them may be specific in prediction of some $\psi$, e.g. $\mu(\psi \mid \phi) > \mu(\psi \mid \neg \phi)$ or $\mu(\psi \mid \phi) < \mu(\psi \mid \neg \phi)$, where $\mu(\phi) > \mu(\neg \phi)$, for instance.
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Let $\mathcal{L}$ be a first-order language of a finite signature.

Allow literals (not only atoms) to appear in a classical logic programming structures of rule, fact and query; denote corresponding sets as $\text{Rule}_\mathcal{L}$, $\text{Fact}_\mathcal{L}$ and $\text{Query}_\mathcal{L}$.

**definition**

Binary relation $C_1 \succ C_2$ (read “$C_1$ is more general than $C_2$”) between $C_1 \equiv (A_1 \leftarrow B_1, \ldots, B_n)$, $C_2 \equiv (A_2 \leftarrow D_1, \ldots, D_m)$ in $\text{Rule}_\mathcal{L}$ takes place iff there exist a substitution $\theta$ such that $\{B_1\theta, \ldots, B_n\theta\} \subseteq \{D_1, \ldots, D_m\}$, $A_1\theta \equiv A_2$ and $\nvdash C_1 \equiv C_2$. 
• Let $\mathcal{G}^*$ be a countable class of observed first-order structures appearing in practice; $\mathcal{G} \subset \mathcal{G}^*$ is a general sampling consisting of well-studied models.

• Being given $\mathcal{G}$ we compute a probability measure $P$ over $\mathcal{G}^*$ with some trusting interval value $\varepsilon > 0$ (according to Kolmogorov); here mathematical statistics is applied.

• Assume $\mu(\phi) \iff P(\{ \mathcal{A} \mid \mathcal{A} \models \phi \})$, where $\phi$ is a closed formulae.
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• Assume $\mu (\phi) \equiv P (\{ \mathcal{A} | \mathcal{A} \models \phi \})$, where $\phi$ is a closed formulae.
Let \( \Theta^o \) be a set of all ground substitutions.

Probability of a ground instance of rule is defined as conditional

\[
\mu (A \leftarrow B_1 \land \ldots \land B_n) = \mu (A \mid B_1 \land \ldots \land B_n) = \frac{\mu (A \land B_1 \land \ldots \land B_n)}{\mu (B_1 \land \ldots \land B_n)}
\]

\[\text{Rule}_L^\mu \models \{ C \mid \text{for some } \theta \in \Theta^o \ \text{probability of } C\theta \text{ is determined}\}\]

\[
\underline{\mu}(C) \models \inf \{ \mu (C\theta) \mid \theta \in \Theta^o \ \text{and } C\theta \in \text{Rule}_L^\mu \},
\]

where \( C \in \text{Rule}_L^\mu \)
Fact \_o is a set of ground atoms allowing verification in any B \in \mathcal{G}^*; a complete set of alternatives is

\[
\text{Fact}^*_o = \text{Fact}_o \cup \{\neg A \mid A \in \text{Fact}_o\}
\]
• Best rules can be viewed as a result of so called *semantic $\mu$-prediction* (notion was introduced in works of E.E. Vityaev) of different literals. For each best rule $C$ used in prediction of some ground $H$ we consider all $C\theta$ such that $\theta$ is a ground substitution satisfying the point $(i)$ of definition.

• We denote by $\text{Prdct}_{\mu}^{\theta,0}$ the obtained set of described ground instances (over all literals $H$).

• Data ($\mathcal{B}$) is a set of actual facts for 1-st order model $\mathcal{B} \in \mathcal{G}^*$, i.e. consistent subset of $\text{Fact}_o^*$ (not necessary maximal).

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A set of literals $S$ is called $\mu$-concurred iff $P(\{A | A \models S\}) \neq 0$. 
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A set of literals $S$ is called *$\mu$-concurred* iff $P (\{\mathcal{A} | \mathcal{A} \vdash S\}) \neq 0$. 
Let some ground atom $H$ be semantically $\mu$-predicted by ground instance $C_{pos} \in \text{Prdct}_{\mu,0}^L$ of the best rule $C_1$ ($C_{pos} \equiv C_1\theta_{pos}$), while $\neg H$ is predicted by $C_{neg} \in \text{Prdct}_{\mu,0}^L$ ($C_{neg} \equiv C_2\theta_{neg}$). Then the set of atoms from premises of $C_{pos}$ and $C_{neg}$ is not $\mu$-concurred.

Denote by $\Gamma_\mathcal{B}$ the following set of rules and data:

$$\{ B_1 \land \ldots \land B_n \rightarrow A \mid A \leftarrow B_1 \land \ldots \land B_n \in \text{Prdct}_{\mu,0}^L \} \cup \text{Data}(\mathcal{B})$$

Let $\text{Data}(\mathcal{B})$ be $\mu$-concurred. Then minimal theory containing $\Gamma_\mathcal{B}$ is logically consistent.
theorem

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Let some ground atom \( H \) be semantically \( \mu \)-predicted by ground instance \( C_{\text{pos}} \in \text{Prdct}_{\mu,0} \) of the best rule \( C_1 \) (\( C_{\text{pos}} \equiv C_1 \theta_{\text{pos}} \)), while \( \neg H \) is predicted by \( C_{\text{neg}} \in \text{Prdct}_{\mu,0} \) (\( C_{\text{neg}} \equiv C_2 \theta_{\text{neg}} \)). Then the set of atoms from premises of \( C_{\text{pos}} \) and \( C_{\text{neg}} \) is not \( \mu \)-concurred.

Denote by \( \Gamma_B \) the following set of rules and data
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\left\{ B_1 \land \ldots \land B_n \rightarrow A \mid A \iff B_1 \land \ldots \land B_n \in \text{Prdct}_{\mu,0} \right\} \cup \text{Data(}\mathcal{B}\text{)}
\]

Let \( \text{Data(}\mathcal{B}\text{)} \) be \( \mu \)-concurred. Then minimal theory containing \( \Gamma_B \) is logically consistent.
Thank you for attention.