

- SERIKZHAN BADAEV, *Computable numberings in the hierarchies*.  
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We consider the computable numberings of the families of the sets from any given level of the well-known hierarchies such as arithmetical, hyperarithmetical, analytical and the Ershov hierarchies. Informally, any computable numbering is identified with the sequence of the sets which has an algorithmic procedure of computation uniform in that level. The set of all computable numberings of a given family  $\mathcal{A}$  is preordered by the reducibility relation: a numbering  $\alpha$  is *reducible* to a numbering  $\beta$  if  $\alpha = \beta \circ f$  for some computable function  $f$ . In a very usual way, this preorder induces some quotient structure on the collection of the computable numberings of  $\mathcal{A}$ . This structure is an upper semilattice, and in the theory of numberings it is called Rogers semilattice of  $\mathcal{A}$ . For the classical case of the families of computably enumerable sets, the Rogers semilattices were studied intensively by the fSU mathematicians since 60th, and beginning the last decade they are studied in many countries for more general classes of sets. This is because of the Rogers semilattice of a family may be treated as a mathematical model of all computations of the family in whole with respect to their computable transformations one into another. We intend to give the current state of art in this area in compare with the classical case. The most part of the results were obtained by the speaker due to joint research with his students and many specialists from the different countries.