

- PHILIPPE BALBIANI, TINKO TINCHEV, *Complete axiomatizations of modal logics for region-based theories of space.*

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The aim of our talk is to give new kinds of propositional modal logics suitable for reasoning about regions in region-based theories of space. Their language contains Boolean variables and standard Boolean operations needed for constructing Boolean terms interpreted by regions in region-based models of space. It also contains the symbol C for the contact relation and the symbol \leq for the part-of relation. Atomic formulas are of the form aCb and $a \leq b$ where a and b are Boolean terms. Complex formulas are built from atomic ones by means of propositional connectives. We use two kinds of semantics. The relational semantics is based on Kripke frames whereas the topological semantics is based on topological spaces. We give axiomatization and completeness theorems with respect to both relational and topological semantics for several important RCC-like logics. Among them are the systems BRCC-8 studied by Wolter and Zakharyashev and the propositional version of GRCC of Li and Ying. All of the considered RCC-like logics are decidable. If we identify them with the set of their theorems, then each one is equivalent to one of the systems BRCC-8. The systems like BRCC-8 have been known before only with respect to their topological semantics. The fact that they are complete both to the relational semantics and the topological semantics is quite remarkable. It shows that the logics of discrete (relational) and indiscrete (topological) theory of space are in a sense indistinguishable. Relational semantics is more simple for obtaining metamathematical results because of its similarity with the semantics of ordinary modal logic and the variety of existing methods and tools in this area. The relationship between pointless and point-based aspects of the presented logical formalisms is reflected as follows. In the syntax of our logics we have regions but no points and points are used in the semantics for modelling regions and defining the relations between regions. The reconstruction of the points in the syntax is in the canonical constructions used for the completeness theorems. In the relational semantics, the points are the ultrafilters in the corresponding canonically defined Boolean algebras. In the topological semantics, we imitate and adapt some techniques known from the theory of proximity spaces where the abstract points are some generalizations of ultrafilters.