

► ANTONGIULIO FORNASIERO, *Tame structures*.

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Let K be a definably complete expansion of an ordered field: that is, every definable bounded subset of K has a least upper bound in K . Important examples of definably complete structures are: o-minimal structures, expansions of the real line, and ultra-products of the above structures. For definably complete structures, we can study various notions of “tameness”, which generalize o-minimality; “tame” but not definably complete structures (e.g. weakly o-minimal structures) are outside the scope of this talk.

The first notion is local o-minimality: we ask o-minimality only around each point of K . Much of the theory of o-minimal structures can be generalized to locally o-minimal ones; ultra-products of o-minimal structures are locally o-minimal.

There is a dichotomy in further generalizing local o-minimality:

1. either we ask that the open core of K (that is, the reduct generated by the open definable sets) is locally o-minimal;
2. or we ask that K is d-minimal: that is, every definable subset X of K is the union of an open set and N discrete sets, where the natural number N does not depend on the parameters of X .

Finally, we consider dense pairs of d-minimal structures: while the theory is quite similar to the o-minimal situation, we cannot apply the machinery of lovely pairs, because, if K is d-minimal but not o-minimal, then the algebraic closure inside K does not satisfy the Exchange Property. We conjecture that such a dense pair has d-minimal open core.

All structures considered will be definably Baire: that is, the union of a definable increasing family of nowhere dense subsets of K is not all of K ; this allows us to use techniques from the theory of Baire spaces.