▸ DANA S. SCOTT, *Mixing Modality and Probability.*
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Orlov first [1928] and Go"del later [1933] pointed out the connection between the Lewis System S4 and Intuitionistic Logic. McKinsey and Tarski gave an algebraic formulation and proved completeness theorems for propositional systems using as models topological spaces with the interior operator corresponding to the necessitation modality. Earlier, Tarski and Stone had each shown that the lattice of open subsets of a topological space models intuitionistic propositional logic. Expanding on a suggestion of Mostowski about interpreting quantifiers, Rasiowa and Sikorski used the topological models to model first-order logic. After the advent of Solovey's recasting of Cohen's independence proofs as using Boolean-valued models, topological models for modal higher- order logic were studied by Gallin and others. (This very, very brief history does not attempt to acknowledge legions of other researchers and investigations of logics other than S4.) For Boolean-valued logic, the complete Boolean algebra Meas([0,1])/Null of measurable subsets of the unit interval modulo sets of measure zero gives every proposition a probability. Perhaps not as well known is the observation that the measure algebra also carries a nontrivial S4 modality defined with the aid of the sublattice Open([0,1])/Null of open sets modulo null sets. This sublattice is closed under arbitrary joins and finite meets in the measure algebra, but it is not the whole of the measure algebra. Consideration of this model of modality brings up several questions:

(1) What completeness results can be expected in the first- order case?

(2) How does this model differ from models used by Montague and Gallin for higher-order logic?

(3) In employing this model to interpret notions of extensional and intensional functions, what revision of the definition of a topos is appropriate?

(4) What kind of definition of random real number should be chosen to go along with the inherent probability?

(5) Will the measure-preserving automorphisms of the modal measure algebra give us a connection between properties of the logic and the results of Ergodic Theory?