The relationship between the Singular Cardinal Hypothesis, Jensen’s square principle, very good scales and large cardinals is important in singular cardinal arithmetic and in understanding how much the universe resembles \(L\). Jensen showed that square holds in \(L\). On the other hand, weak square fails above a supercompact, and implies that every scale is good.

There have also been results about singular cardinals that are not relative consistency results. Using PCF theory Shelah showed that if \(2^{\aleph_n} < \aleph_{\omega} \) for every \(n < \omega\), then \(2^{\aleph_\omega} < \aleph_{\omega+4}\). Scales are a central concept in PCF theory and are very useful in exploring the tension between combinatorial principles like square and the reflection properties in the presence of large cardinals.

We will discuss relative consistency results about the relationship between these principles in the context of forcing and large cardinals.