A theory \( T \) is called geometric if in all of its models algebraic closure satisfies the exchange property, and \( T \) eliminates the quantifier \( \exists^\infty \). The class of geometric theories includes both \( \omega \)-minimal theories (extending DLO) and strongly minimal theories, where there is a clear dividing line between linear and non-linear structures, with linearity characterized by various equivalent conditions (e.g. the CF property and non-interpretability of infinite fields in the \( \omega \)-minimal case, 1-basedness and local modularity in the strongly minimal case). Situation is more complicated in the SU-rank 1, thorn rank 1 and geometric cases, e.g. in the SU-rank 1 case local modularity is stronger than 1-basedness.

We analyze linearity-like conditions using lovely pairs of geometric structures, a common generalization of lovely (generic) pairs of supersimple SU-rank 1 structures [2] and dense pairs of \( \omega \)-minimal structures [4]. We call a geometric theory weakly locally modular, if in a lovely pair \((M, P)\) of its models the pregeometry \((M, acl(- \cup P(M)))\) of the localization of \( acl \) at \( P(M) \) is modular. This property coincides with 1-basedness in the SU-rank 1 case, and with the CF property in the \( \omega \)-minimal case, and implies that the geometry of \((M, acl(- \cup P(M)))\) is a disjoint union of projective geometries over division rings and/or a trivial geometry. It is shown in [2, 3] that for an SU-rank 1 theory \( T \), \( TP \) is supersimple of SU-rank 1, 2 or \( \omega \). Gareth Boxall [1] has established superrosiness of \( TP \) for a superrosy theory \( T \) of thorn-rank 1. We show that if, in addition, \( T \) is weakly locally modular, then \( TP \) has thorn-rank \( \leq 2 \). Using the Trichotomy theorem we show that for an \( \omega \)-minimal theory \( T \) extending DLO, \( TP \) is superrosy of thorn-rank 1, 2 or \( \omega \).

This is a joint work with Alexander Berenstein.