▶ A. ABAJYAN, A. CHUBARYAN, *Proof complexity of hard-determinable formulas in $R(lin)$*.
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The system $R(lin)$ (Ran Raz and Iddo Tzameret) is an extension of propositional resolution system by allowing it to operate with disjunctions of linear equations instead of clauses. The authors proved that well-known hard tautologies (PHP_n^m, $\mathrm{Clique}_{n,k}$, Tseitin tautologies) have polynomial-size $R(lin)$-proofs. Earlier by second author of this abstract the concept of hard-determinable formulas was introduced and it was proved, that the proof complexity of such formulas has exponential lower bounds in "weak" proof systems, but the property of hard-determinability is insufficient for obtaining a superpolynomial lower bound of Frege proof complexities. The above mentioned hard formulas are not hard-determinable and therefore of great interests is the investigation of $R(lin)$-proof complexities just for hard-determinable formulas.

Let φ be a propositional formula, let $P = \{p_1, p_2, \dots, p_n\}$ be the set of the distinct variables of φ and let $P' = \{p_{i_1}, p_{i_2}, \dots, p_{i_m}\}$ $(1 \leqslant m \leqslant n)$ be some subset of P.

Given $\sigma = \{\sigma_1, \dots, \sigma_m\} \in E^m$, the conjunct $K^\sigma = \left\{p_{i_1}^{\sigma_1}, p_{i_2}^{\sigma_2}, \dots, p_{i_m}^{\sigma_m}\right\}$ is called *φ-determinative* if assigning σ_j $(1 \leqslant j \leqslant m)$ to each p_{i_j} we obtain the value of φ (0 or 1) independently of the values of the remaining variables.

The minimal possible number of variables in a φ-determinative conjunct is denoted by $d(\varphi)$.

Let φ_n $(n \geqslant 1)$ be a sequence of minimal tautologies and $|\varphi_n|$ be the size of φ_n. If for some n_0 there is a constant c such that $\forall n \geqslant n_0$ $(d(\varphi_n))^c \leqslant |\varphi_n| < (d(\varphi_n))^{c+1}$, then the formulas $\varphi_{n_0}, \varphi_{n_0+1}, \varphi_{n_0+2}, \dots$ are *hard-determinable*.

Let
$$\varphi_n = \bigvee_{(\sigma_1,\dots,\sigma_n)\in E^n} \mathop{\&}_{j=1}^{2^n-1} \bigvee_{i=1}^{n} p_{ij}^{\sigma_i} \qquad (n \geqslant 1).$$
It is not difficult to see, that the formulas $\varphi_3, \varphi_4, \dots$ are hard-determinable.

We prove the following statement

Theorem. *There is a polynomial-size $R(lin)$ refutation of $\neg\varphi_n$.*