

- ▶ PAUL SHAFER, *The first order theory of the Medvedev lattice is third order arithmetic.* Department of Mathematics, Cornell University, 310 Malott Hall, Ithaca NY, 14853, USA.

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For sets $\mathcal{A}, \mathcal{B} \subseteq \omega^\omega$, we say \mathcal{A} *Medvedev reduces* to \mathcal{B} ($\mathcal{A} \leq_M \mathcal{B}$) if there is a Turing functional Φ such that Φ^f is total and in \mathcal{A} for all f in \mathcal{B} , and we say $\mathcal{A}, \mathcal{B} \subseteq \omega^\omega$ are *Medvedev equivalent* ($\mathcal{A} \equiv_M \mathcal{B}$) if $\mathcal{A} \leq_M \mathcal{B}$ and $\mathcal{B} \leq_M \mathcal{A}$. The *Medvedev lattice* is the degree structure $(\mathcal{P}(\omega^\omega)/\equiv_M, \leq)$, where \leq is the ordering \leq_M induces on equivalence classes. Similarly, \mathcal{A} *weakly reduces* to \mathcal{B} ($\mathcal{A} \leq_w \mathcal{B}$) if for every f in \mathcal{B} there is g in \mathcal{A} with $f \geq_T g$. *Weak equivalence* (\equiv_w) and the *Muchnik lattice* $(\mathcal{P}(\omega^\omega)/\equiv_w, \leq)$ are defined analogously to Medvedev equivalence and the Medvedev lattice. We code the standard model of third order arithmetic directly into the Muchnik lattice. This result, combined with the definability of the Muchnik lattice in the Medvedev lattice and the definability of the Medvedev lattice in true third order arithmetic, proves the following theorem: the first order theory of the Muchnik lattice, the first order theory of the Medvedev lattice, and the third order theory of true arithmetic are recursively isomorphic. This approach is different from the one taken by Lewis, Nies, and Sorbi at CiE 2009.