

- TAPANI HYTTINEN, VADIM KULIKOV, *Weak Ehrenfeucht-Fraïssé equivalences.*
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A *Weak Ehrenfeucht-Fraïssé game on L -structures \mathcal{A} and \mathcal{B} of length α* , denoted by $EF_\alpha^*(\mathcal{A}, \mathcal{B})$ or shorter, is played between two players I and II on two L -structures \mathcal{A} and \mathcal{B} , where L is a relational vocabulary. The players choose elements of the domains of the structures in α moves, and in the end of the game the player II wins if the chosen structures are isomorphic. Otherwise player I wins.

The obvious difference of this to the ordinary Ehrenfeucht-Fraïssé game is that the isomorphism can be arbitrary whereas in the ordinary EF-game it should be determined by the moves of the players. In particular this game is not closed (in the sense of Gale-Stewart [3]). In our article we answer the following questions and in the talk we discuss some of them.

- Are the games EF_ω and EF_ω^* equivalent? This was solved already by Kueker in [1] in the context of cub-subsets of power sets. (Answer: yes)
- Are the games EF_α and EF_α^* equivalent for an ordinal α ? (Answer: no)
- Are the games EF_κ and EF_κ^* always equivalent for a cardinal $\kappa > \omega$? (Answer: for structures of size κ^{++} no, for $\kappa = \omega_1$ and structures of size \aleph_2 , independent of ZFC. Here we use results of [2])
- If structures are weakly α -equivalent and $\beta < \alpha$, are they necessarily weakly β -equivalent? (Answer: no)
- Is $EF_{\omega_1}^*$ necessarily determined? (Answer: independent of ZFC, if the size of the structures is \aleph_2 and the answer is no, if the size of the structures is greater than \aleph_2).

[1] D. W. Kueker *Countable approximations and Löwenheim-Skolem theorems*, *Annals of Math. Logic*, 11 (1977) 57-103.

[2] A. H. Mekler, S. Shelah and J. Väänänen: *The Ehrenfeucht-Fraïssé-game of length ω_1* , *Transactions of the American Mathematical Society*, 339:567-580, 1993., 11 (1977) 57-103.

[3] Gale, D. and Stewart, F. M. *Infinite games with perfect information*. In *Contributions to the theory of games*, vol. 2, *Annals of Mathematics Studies*, no. 28, pages 245–266. Princeton University Press, Princeton, N. J., 1953.