

- HORACIO FAAS, *Between geometry and arithmetic: the long journey to the calculus*.  
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Visualization played an important role in ancient mathematics, what be illustrated by the discovery of irrational numbers, vg. square root of two, which even though non expressible by the then known numbers could be immediately "seen" as the diagonal of the square of side equal one. As we all know, straightedge and compass were the only tools admitted by ancient Greek geometers for the construction of their objects of inquiry. With such tools they were able to square any figure with straight sides, but the squaring of curved lines remained highly problematic. Nonetheless, Hippocrates of Chios (ca. 440 B.C.) succeeded in squaring a "lune", an area bounded by arcs of circles. This result encouraged him and other mathematicians at the time to try to resolve the squaring of the circle. But all attempts to solve the problem failed, even though the reason for this failure could only be established in 1882 by the proof of the transcendence of pi due to Lindemann.

It is often claimed that ancient Greek geometry remained attached to figures and shapes. But how fundamental was the role of geometric visualization in the modern advancement towards the calculus of areas? In my paper, I intend to show that even by the time the naive approach of ancient Greek geometry was abandoned, leading mathematicians working on the development of the calculus, including one of its founders, Leibniz, and later on, Cauchy were indeed inspired by visual insights.

In this paper I aim to highlight the role of visualization along the road to the calculus by discussing some aspects of Hippocrates's work and concluding with references to Leibniz's Transmutation Theorem, Cauchy's definition of the integral and Liouville's first transcendental number.