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A quantified modal logic for rough sets.

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A structure of the form $(U, \{R_i\}_{i\in N})$, where N is an initial segment of $\aleph$ representing N 'sources' and R_i is an equivalence relation on U representing the knowledge base of the i^{th} source, was considered in [3] to formally study the behavior of rough sets [4] in a multiple-source scenario. In this paper, we extend our study to consider structures of the form $(U, \{R_P\}_{P\subseteq_f N})$, called *multiple-source approximation systems for groups* (denoted $MSAS^G$), where R_P represents the combined knowledge base of the finite group P of sources. R_P is an equivalence relation on U satisfying (i) $R_P = \cap_{i\in P} R_i$ and (ii) $R_\emptyset = U\times U$. A quantified propositional modal logic $LMSAS^G$, different from modal logic with propositional quantifiers [1] and modal predicate logic, is proposed with semantics based on $MSAS^G$'s. The language has a set PV of propositional variables, and a set T of terms built with countable sets of constants and variables and a binary function symbol $\star$. Formulae are got through the scheme: $p\,|\neg\alpha|\alpha\wedge\beta|A\alpha|[t]\alpha|t = s|\forall x\alpha$, where $p \in PV$, $t, s \in T$, and A is the global modal operator. Thus quantification ranges over modalities. The semantics is defined with the help of a function which maps a term t to a finite subset of N, $*$ being translated as union of sets. The function determines which equivalence relation is to be used to evaluate a modality involving a term t. A sound and complete axiomatization is given and some decidability problems are addressed. It is found that the modal systems $B, S5$ and epistemic logics $S5_n^D$ [2] are embedded in $LMSAS^G$. It is also observed that $S5_n^D$ cannot replace $MSAS^G$ to serve our purpose. The semantics of $S5_n^D$ considers a finite and fixed number of agents, thus giving a finite and fixed number of modalities in the language. But in the case of $LMSAS^G$, the number of sources is not fixed, and could also be countably infinite. So, unlike the case of epistemic logics, it is not possible here to refer to all/some sources using only the connectives $\wedge, \vee$. The quantifiers $\forall, \exists$ are used to achieve the task.

[1] R. A. Bull, *On modal logic with propositional quantifiers*, ***J. Symbolic Logic***, vol. 34 (1969), no. 2, pp. 257–263.

[2] R. Fagin, J.Y. Halpern, M.Y. Vardi, ***Reasoning About Knowledge***, The MIT Press, Dordrecht, 1995.

[3] M. A. Khan, M. Banerjee, *Formal reasoning with rough sets in multiple-source approximation systems*, ***Int. J. Approximate Reasoning***, vol. 49 (2008), pp. 466–477.

[4] Z. Pawlak, ***Rough Sets. Theoretical Aspects of Reasoning about Data***, Kluwer Academic Publishers, Dordrecht, 1991.