▶ PHILIPPE BALBIANI, *Lexicographic products of modal logics with linear frames.*

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Given Kripke complete modal logics $L_1, \ldots, L_n$ in languages, respectively based on $\Box_1, \ldots, \Box_n$ the product of $L_1, \ldots, L_n$ is defined as the multimodal logic $L_1 \times \cdots \times L_n = Log\{\mathcal{F}_1 \times \cdots \times \mathcal{F}_n : \mathcal{F}_1 \models L_1, \ldots, \mathcal{F}_n \models L_n\}$ in a language based on $\Box_1, \ldots, \Box_n$, where the product $\mathcal{F}_1 \times \cdots \times \mathcal{F}_n$ of structures $\mathcal{F}_1 = (W_1, R_1), \ldots, \mathcal{F}_n = (W_n, R_n)$ is the structure $\mathcal{F} = (W, S_1 \ldots, S_n)$ defined by putting $W = W_1 \times \cdots \times W_n$ and $(x_1, \ldots, x_n)S_i(y_1, \ldots, y_n)$ iff $x_1 = y_1, \ldots, x_{i-1} = y_{i-1}, x_iR_iy_i, x_{i+1} = y_{i+1}, \ldots, x_n = y_n$ for each $x_1, y_1 \in W_1, \ldots, x_n, y_n \in W_n$ and for each $i \in \{1, \ldots, n\}$. The above product $\mathcal{F}_1 \times \cdots \times \mathcal{F}_n$ of structures has been considered within the context of reasoning about the knowledge [3]. See [5] for a detailed study of the axiomatization and the decidability of the modal logic it gives rise to.

Given Kripke complete modal logics $L_1, \ldots, L_n$ in languages, respectively based on $\Box_1, \ldots, \Box_n$, it is also make sense to consider the "lexicographic" product of $L_1, \ldots, L_n$ defined as the multimodal logic $L_1 \rhd \cdots \rhd L_n = Log\{\mathcal{F}_1 \rhd \cdots \rhd \mathcal{F}_n : \mathcal{F}_1 \models L_1, \ldots, \mathcal{F}_n \models L_n\}$ in a language based on $\Box_1, \ldots, \Box_n$, where the product $\mathcal{F}_1 \rhd \cdots \rhd \mathcal{F}_n$ of structures $\mathcal{F}_1 = (W_1, R_1), \ldots, \mathcal{F}_n = (W_n, R_n)$ is the structure $\mathcal{F} = (W, S_1 \ldots, S_n)$ defined by putting $W = W_1 \times \cdots \times W_n$ and $(x_1, \ldots, x_n)S_i(y_1, \ldots, y_n)$ iff $x_iR_iy_i, x_{i+1} = y_{i+1}, \ldots, x_n = y_n$ for each $x_1, y_1 \in W_1, \ldots, x_n, y_n \in W_n$ and for each $i \in \{1, \ldots, n\}$. The above product $\mathcal{F}_1 \rhd \cdots \rhd \mathcal{F}_n$ of structures has been considered within the context of reasoning about the time [1]. The detailed study of the axiomatization and the decidability of the modal logic it gives rise to has never been undertaken.

In this talk, we will study the "lexicographic" products of Kripke complete modal logics $L_1, \ldots, L_n$ containing $S4 \cdot 3$ and we will prove that these modal logics have the finite frame property, are finitely axiomatizable and are $coNP$-complete.

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