Low linear orderings

Andrey N. Frolov

Kazan State University
Kazan, Russia
Andrey.Frolov@ksu.ru

August 5, 2009

Low linear orderings

Theorem (C.G. Jockusch and R.I. Soare, 1991)

Every noncomputable c.e. degree contains linear ordering with no computable copy.

Corollary

There exists a low linear ordering with no a computable copy.

Definition

X is low if $X' \leq_T \emptyset'$,
X is low$_n$ if $X^{(n)} \leq_T \emptyset^{(n)}$.

Low linear orderings

Theorem (R.G. Downey and M.F. Moses, 1989)

Any low discrete linear ordering has a computable copy.

Definition

A linear ordering is called *discrete* if every element is contained into a segment which is isomorphic to $\omega^* + \omega$.

Question (R. Downey)

What is a property P of order types which guarantees that if a linear ordering L is low and $P(L)$ then L has a computable copy?

Low linear orderings

Definition (Fr., 2006)

A linear ordering is called *quasidiscrete* if any pair of successors is contained into a segment which is isomorphic to ω^* or ω.

Theorem (P. Alaev, Fr., and J. Thurber, submitted; Fr., 2006)

- Any low_2 quasidiscrete linear ordering is Δ^0_4-isomorphic to a computable one.
- If the ordering is discrete then the isomorphism is Δ^0_3.
- If the ordering is low then complexities of the isomorphisms are the same.
- There exists a low_3 discrete linear ordering with no a computable copy.

Low linear orderings

Definition

- A linear ordering L is called *η-like* if $L \cong \sum_{q \in \mathbb{Q}} f(q)$, where $f : \mathbb{Q} \to \omega - \{0\}$.
- If the f is bounded then the ordering is called *strongly η-like*.

Theorem (Fr., 2006)

- Every low strongly η-like linear ordering is Δ^0_3-isomorphic to a computable one.
- There exists a low_2 strongly η-like linear ordering with no a computable copy.

Theorem (Fr. and M. Zubkov, 2009; after Fr., 2006)

If L is η-like linear ordering then L has a Δ^0_2 copy with Δ^0_2 successor and block relations iff L has a computable copy with a Π^0_1 block relation.

Corollary (Fr., 2006)

Any low strongly η-like linear ordering has a computable copy.

Low linear orderings

Definition (Fr., submitted)

A linear ordering L is called *k-quasidiscrete* if for any element x either $|[x]_L| \leq k$ or $|[x]_L| = +\infty$, where $[x]_L = \{y \mid [x, y]_L$ or $[y, x]_L$ is finite$\}$.

Theorem (Fr., submitted)

- Any low k-quasidiscrete linear ordering is Δ^0_4-isomorphic to a computable one.
- If the ordering is discrete then the isomorphism is Δ^0_3.
- If the ordering is strongly η-like then the isomorphism is Δ^0_2.
- In each case, the isomorphism has no better level of the arithmetical hierarchy.

Theorem (Fr., submitted)

Any Δ^0_2 linear ordering with a Δ^0_2 successor relation is Δ^0_2-isomorphic to a low one.

Corollary (Fr., ?)

A linear ordering $(\omega, <_L)$ has a low copy iff $(\omega, <_L, S_L)$ has a Δ^0_2 copy.

Where $S_L(x, y)$ is the successor relation on L.

Low linear orderings

Theorem (Fr., in preparation)

Every low linear ordering L has a low copy M such that M is not Δ^0_1-isomorphic to a computable one.

Theorem (Fr., in preparation)

Every low linear ordering L, which is not strongly η-like, has a low copy M such that M is not Δ^0_2-isomorphic to a computable one.

Δ^0_n-version of Downey's question

What is a property P_n of order types which guarantees that if a linear ordering L is low and $P_n(L)$ then L is Δ^0_n-isomorphic to a computable one?

Thank you
See you on LC'2010