Computational intension, denotation and propositional intention in the languages of acyclic recursion

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Logic Colloquium 2009
Sofia

August 1, 2009

- Moschovakis (2003–2006): L_{ar}^{λ} and L_{r}^{λ} have twofold semantics:

$$\underbrace{\text{Syntax of } L_{ar}^{\lambda}\ (L_{r}^{\lambda}) \Longrightarrow \text{Referential Intensions (Algorithms)} \Longrightarrow \text{Denotations}}_{\text{Semantics of } L_{ar}^{\lambda}(L_{r}^{\lambda})}$$

- Applications of L_{ar}^{λ}
  - computational semantics of NL
    - Antecedent–anaphora relations
    - Quantifier scope underspecification
  - syntax-semantics interface for NLP
- $L_{ar}^{\lambda} \subset L_{exar}^{\lambda}$
  (Montague (70s) $\prec$ Thomason (1980) $\prec$ Muskens (2005))

  For each propositional term $A : p$
  1. den(A): *T-intention*
     the propositional denotation of A (per se)
  2. den($\mathcal{E}(A)$):
     truth-functional denotation, the extension of A (int., the set of all states in which den(A) holds)

Languages of Acyclic Recursion: L^{λ}_{ar} / L^{λ}_{exar}

Propositions vs. Extensions of Propositions Ideas from

- T-intentional Logic of Propositions:
  ideas by Thomason (1980), Muskens (2005)

The set *Types* of L^{λ}_{ar} / L^{λ}_{exar}:

$$\sigma :\equiv e \mid t \mid p \mid s \mid (\tau_1 \to \tau_2) \qquad (Types)$$

Some abbreviations:

- $\widetilde{\mathrm{e}} \equiv (\mathrm{s} \to \mathrm{e})$ (the type of individual concepts)
- $\widetilde{\mathrm{t}} \equiv (\mathrm{s} \to \mathrm{t})$ (the type of state extensions of propositions)
- $\widetilde{\mathrm{p}} \equiv (\mathrm{s} \to \mathrm{p})$ (the type of *situated propositions*)
- $\widetilde{\tau} \equiv (\mathrm{s} \to \tau)$, where $\tau \in Types$
- $\tau_1 \times \tau_2 \to \tau \equiv (\tau_1 \to (\tau_2 \to \tau))$, where $\tau_1, \tau_2, \tau \in Types$

Syntax of L^{λ}_{ar} and L^{λ}_{exar}

- Constants: $K_\tau = \{\mathrm{c}^\tau_0, \ldots, \mathrm{c}^\tau_{k_\tau}\}$
  - a special constant, $\mathcal{E} \in K_{\widetilde{\mathrm{p}} \to \widetilde{\mathrm{t}}}$:
    $\mathcal{E}(P) : \widetilde{\mathrm{t}}$ — the set of states (that have information) validating $P : \widetilde{\mathrm{p}}$
- $PureVars_\tau = \{v^\tau_0, v^\tau_1, \ldots\}$, $RecVars_\tau = \{p^\tau_0, p^\tau_1, \ldots\}$
- Terms of L^{λ}_{ar} (L^{λ}_{exar})

$$A :\equiv \mathrm{c}^\tau : \tau \mid x^\tau : \tau \mid B^{(\sigma\to\tau)}(C^\sigma) : \tau \mid \lambda v^\sigma\, (B^\tau) : (\sigma \to \tau)$$
$$\mid A^\sigma_0 \text{ where } \{p^{\sigma_1}_1 := A^{\sigma_1}_1, \ldots, p^{\sigma_n}_n := A^{\sigma_n}_n\} : \sigma$$

where $\{p_1 := A_1, \ldots, p_n := A_n\}$ is an acyclic system

i.e., there is a function $rank : \{p_1, \ldots, p_n\} \longrightarrow \mathbb{N}$ such that, for all $i, j \in \{1, \ldots, n\}$:

if p_j occurs free in A_i, then $rank(p_j) < rank(p_i)$.

Canonical Form Theorem: For each term A, there is a unique, up to congruence, irreducible term denoted by $\mathrm{cf}(A)$, such that:

1. $\mathrm{cf}(A) \equiv A$ or $\mathrm{cf}(A) \equiv A_0 \text{ where } \{p_1 := A_1, \ldots, p_n := A_n\}$
2. $A \Rightarrow \mathrm{cf}(A)$

Referential Synonymy Theorem: Two terms A, B are referentially synonymous, $A \approx B$, iff there are explicit, irreducible terms (of appropriate types), $A_0, A_1, \ldots, A_n, B_0, B_1, \ldots, B_n$, $n \geq 0$, such that:

- $A \Rightarrow_{cf} A_0 \text{ where } \{p_1 := A_1, \ldots, p_n := A_n\}$,
- $B \Rightarrow_{cf} B_0 \text{ where } \{p_1 := B_1, \ldots, p_n := B_n\}$,
- $\models A_i = B_i$ $(i = 0, \ldots, n)$, i.e., $\mathrm{den}(A_i)(g) = \mathrm{den}(B_i)(g)$ for all variable assignments g.

NL Category	L^{λ}_{exar} Constants	L^{λ}_{exar} Type
PureObj	$0, 1, 2, \ldots$	e
NP	$\mathit{john}, \mathit{mary}, \ldots$	$\widetilde{\mathrm{e}}$
IV	$\mathit{run}, \mathit{smile}, \ldots$	$(\widetilde{\mathrm{e}} \rightarrow \widetilde{\mathrm{p}})$
CN	$\mathit{man}, \mathit{woman}, \mathit{dog}, \ldots$	$(\widetilde{\mathrm{e}} \rightarrow \widetilde{\mathrm{p}})$
TV	$\mathit{like}, \mathit{love}, \ldots$	$(\widetilde{\mathrm{e}} \rightarrow (\widetilde{\mathrm{e}} \rightarrow \widetilde{\mathrm{p}}))$
ATV	$\mathit{believe}, \mathit{knows}, \ldots$	$(\widetilde{\mathrm{e}} \rightarrow (\widetilde{\mathrm{p}} \rightarrow \widetilde{\mathrm{p}}))$
QNP	$\mathit{everything}, \mathit{something}$	$((\widetilde{\mathrm{e}} \rightarrow \widetilde{\mathrm{p}}) \rightarrow \widetilde{\mathrm{p}})$
Det	$\mathit{every}, \mathit{some}, \mathit{a}, \ldots$	$(\widetilde{\mathrm{e}} \rightarrow \widetilde{\mathrm{p}}) \times (\widetilde{\mathrm{e}} \rightarrow \widetilde{\mathrm{p}}) \rightarrow \widetilde{\mathrm{p}}$
Coord	$\mathit{and}, \mathit{or},\ \mathit{if}$	$((\widetilde{\mathrm{p}} \rightarrow \widetilde{\mathrm{p}}) \rightarrow \widetilde{\mathrm{p}})$
SNeg	not	$(\widetilde{\mathrm{p}} \rightarrow \widetilde{\mathrm{p}})$

Table: Examples of L^{λ}_{exar} constants and types rendering NL expressions and lexemes (words)

Different kinds of antecedent–anaphora relations by L_{ar}^{λ}

- Strict, reflexive anaphora via co-indexing is required in some cases.

For ex., in NL (spoken by humans): the syntax-semantics interface of reflexive pronouns, like "herself", can be regulated by co-indexing arguments, as in options (1a)-(1b), but not by (1c).

$$\text{Mary likes herself.} \xrightarrow{\text{render}} \qquad \text{three options:}$$

$$\lambda x\, like(x, x)(m) \text{ where } \{m := mary\} \qquad (\lambda \text{ co-index}) \quad (1a)$$

$$\approx like(m, m) \text{ where } \{m := mary\} \qquad (\text{ar co-index}) \quad (1b)$$

$$\not\approx like(m_1, m_2) \text{ where } \{m_2 := m_1,\ m_1 := mary\} \qquad (1c)$$

Reflexive vs. irreflexive antecedent–anaphora relations

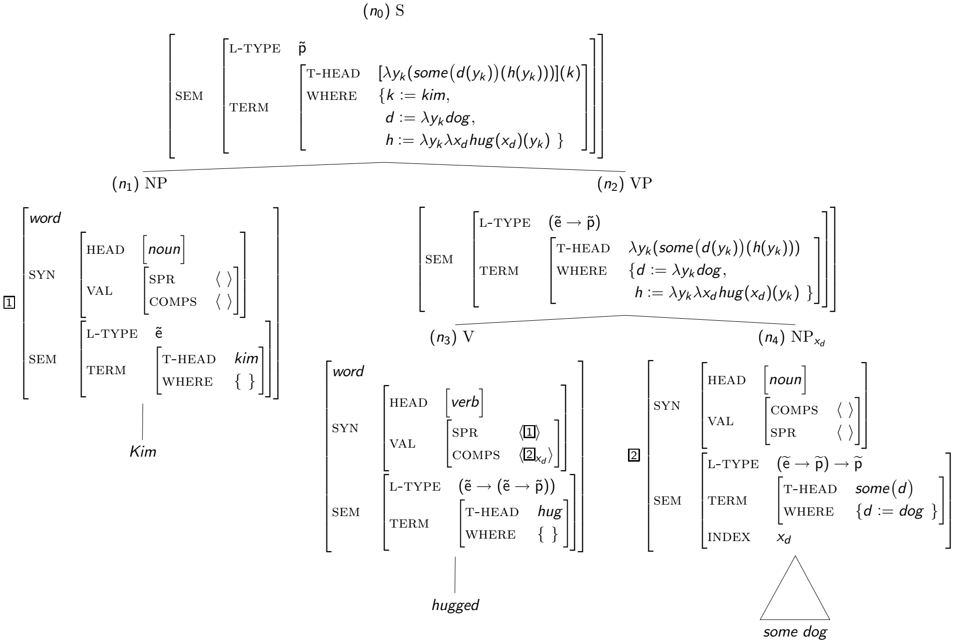
- Co-indexing, as in (2a), is not good for non-reflexive pronouns
- Underspecified arguments: (2b)
- Resolution of underspecification by the context: (2c).

John loves his wife and he honors her. $\xrightarrow{\text{render}}$ options:

$$[L \,\&\, H] \text{ where } \{L := love(j, w),\ H := honors(j, w),\ j := john,\ w := wife(j)\} \quad \text{(2a) (ar co-index)}$$

$$[L \,\&\, H] \text{ where } \{L := love(j, w),\ H := honors(h_1, h_2),\ j := john,\ w := wife(j)\} \quad \text{(2b) (underspec)}$$

$$[L \,\&\, H] \text{ where } \{L := love(j, w),\ H := honors(h_1, h_2),\ h_1 := j,\ j := john,\ h_2 := w,\ w := wife(j)\} \quad \text{(2c) (no } \lambda\text{-term)}$$



(n0) S
SEM
L-TYPE p̃
TERM
T-HEAD [λy_k(some(d(y_k))(h(y_k)))](k)
WHERE {k := kim, d := λy_k dog, h := λy_k λx_d hug(x_d)(y_k) }
(n1) NP
1
word
SYN
HEAD noun
VAL
SPR ⟨ ⟩
COMPS ⟨ ⟩
SEM
L-TYPE ẽ
TERM
T-HEAD kim
WHERE { }
Kim
(n2) VP
SEM
L-TYPE (ẽ → p̃)
TERM
T-HEAD λy_k(some(d(y_k))(h(y_k)))
WHERE {d := λy_k dog, h := λy_k λx_d hug(x_d)(y_k) }
(n3) V
word
SYN
HEAD verb
VAL
SPR ⟨1⟩
COMPS ⟨2 x_d⟩
SEM
L-TYPE (ẽ → (ẽ → p̃))
TERM
T-HEAD hug
WHERE { }
hugged
(n4) NP x_d
2
SYN
HEAD noun
VAL
COMPS ⟨ ⟩
SPR ⟨ ⟩
SEM
L-TYPE (ẽ → p̃) → p̃
TERM
T-HEAD some(d)
WHERE {d := dog }
INDEX x_d
some dog

$$\text{John likes Mary's father.} \tag{3a}$$

$$\xrightarrow{\text{render}} \textit{like}(\textit{john})(\textit{father\_of}(\textit{mary})) \ : \ \widetilde{\mathrm{p}} \tag{3b}$$

$$\Rightarrow_{cf} \textit{like}(j)(f) \,\text{where}\, \{j := \textit{john},\ m := \textit{mary},\ f := \textit{father\_of}(m)\} \tag{3c}$$

$$\mathcal{E}(\textit{like}(\textit{john})(\textit{father\_of}(\textit{mary}))) \ : \ \widetilde{\mathrm{t}} \tag{4a}$$

$$\Rightarrow \mathcal{E}(P) \,\text{where}\, \{P := \textit{like}(\textit{john})(\textit{father\_of}(\textit{mary}))\} \tag{4b}$$

$$\Rightarrow_{cf} \mathcal{E}(P) \,\text{where}\, \{P := \textit{like}(j)(f),\ j := \textit{john},\ m := \textit{mary},\ f := \textit{father\_of}(m)\} \tag{4c}$$

Informally: For any $d : s$,

$$\mathrm{den}(\mathcal{E}(P)(d)) = 1 \text{ iff the proposition } \mathrm{den}(\textit{like}(j)(f)(d)) \text{ holds} \tag{5}$$

$$\text{iff in } \mathrm{den}(d), \text{ the situated prop. } \mathrm{den}(\textit{like}(j)(f)) \text{ is true} \tag{6}$$

One more clause to the definition of Canonical Forms

For every $A : \widetilde{\mathrm{p}}$, such that
$\mathrm{cf}(A) \equiv A_0 \text{ where } \{p_1 := A_1, \dots, p_n := A_n\}$,

$$\mathrm{cf}(\exists x A) :\equiv \exists x p(x) \text{ where } \{p := \lambda x A'_0, p'_1 := \lambda x A'_1, \dots, p'_n := \lambda x A'_n\} \tag{7a}$$

$$\mathrm{cf}(\forall x A) :\equiv \forall x p(x) \text{ where } \{p := \lambda x A'_0, p'_1 := \lambda x A'_1, \dots, p'_n := \lambda x A'_n\} \tag{7b}$$

where for all $i = 1, \dots, n$, p'_i is a fresh location, and
for all $i = 0, \dots, n$, $A'_i :\equiv A_i\{p_1 :\equiv p'_1(x), \dots, p_n :\equiv p'_n(x)\}$.

Let $C \in \{and,\ or,\ if,\ \}$,
$Q, Q_i : \widetilde{\mathrm{p}}$, $\quad \mathrm{cf}(Q) = Q_0 \,\text{where}\, \{\vec{q} := \vec{Q}\}$, and
$\mathrm{cf}(Q_i) = Q_{i,0} \,\text{where}\, \{\vec{q_i} := \vec{Q_i}\}$, for $i \in \{1, 2\}$. By the def of the canonical forms:

$$\begin{aligned}
\mathrm{cf}(\mathcal{E}(C(Q_1, Q_2))) \equiv \mathcal{E}(Q) \,\text{where}\, \{ & Q := C(q_1, q_2), \\
& q_1 := Q_{1,0}, q_2 := Q_{2,0}, \\
& \vec{q_1} := \vec{Q_1}, \vec{q_2} := \vec{Q_2}\}
\end{aligned} \tag{8a}$$

$$\begin{aligned}
\mathrm{cf}(\mathcal{E}(not(Q)) \equiv \mathcal{E}(N) \,\text{where}\, \{ & N := not(q), q := Q, \\
& \vec{q} := \vec{Q}\}
\end{aligned} \tag{8b}$$

$$\mathrm{cf}(\mathcal{E}(\exists x Q)) \equiv \mathcal{E}(N) \,\text{where}\, \{N := \exists x Q_0, \vec{q} := \vec{Q}\} \tag{8c}$$

$$\mathrm{cf}(\mathcal{E}(\forall x Q)) \equiv \mathcal{E}(N) \,\text{where}\, \{N := \forall x Q_0, \vec{q} := \vec{Q}\} \tag{8d}$$

By (8a)-(8d), the truth evaluation by $\mathcal{E}$ doesn't proceed compositionally through the propositional sub-terms of the logical connectors.

Definition

$$\mathcal{E}(not(X)) \Rightarrow_T \neg(p) \text{ where } \{p := \mathcal{E}(X)\} \tag{9a}$$

$$\mathcal{E}(and(X_1, X_2)) \Rightarrow_T (p_1 \,\&\, p_2) \text{ where } \{p_1 := \mathcal{E}(X_1), p_2 := \mathcal{E}(X_2)\} \tag{9b}$$

$$\mathcal{E}(or(X_1, X_2)) \Rightarrow_T (p_1 \vee p_2) \text{ where } \{p_1 := \mathcal{E}(X_1), p_2 := \mathcal{E}(X_2)\} \tag{9c}$$

$$\mathcal{E}(if(X_1, X_2)) \Rightarrow_T (p_1 \rightarrow p_2) \text{ where } \{p_1 := \mathcal{E}(X_1), p_2 := \mathcal{E}(X_2)\} \tag{9d}$$

$$\begin{aligned}\mathcal{E}(some(X_1, X_2)) \Rightarrow_T \exists x(p_1(x) \,\&\, p_2(x)) \text{ where } \{&p_1 := \lambda x \mathcal{E}(X_1(x)),\\ &p_2 := \lambda x \mathcal{E}(X_2(x)) \quad (x \text{ is fresh})\end{aligned} \tag{9e}$$

$$\begin{aligned}\mathcal{E}(every(X_1, X_2)) \Rightarrow_T \forall x(p_1(x) \rightarrow p_2(x)) \text{ where } \{&p_1 := \lambda x \mathcal{E}(X_1(x)),\\ &p_2 := \lambda x \mathcal{E}(X_2(x)) \quad (x \text{ is fresh})\end{aligned} \tag{9f}$$

$$\begin{aligned}\mathcal{E}(is(X_1, X_2)) \Rightarrow_T \lambda d(p_1(d) = p_2(d)) &\text{ where}\\ &\{p_i := \lambda d X_i(d) \mid i \in \{1, 2\}\}(d : s, \text{ is fresh})\end{aligned} \tag{9g}$$

Note: The def. should have cases w.r.t. the immediate terms.

Definition

For any non-logical constant $R : (\sigma_1 \times \ldots \times \sigma_n \to \widetilde{\mathrm{p}})$ and any immediate terms $X_1 : \sigma_1, \ldots, X_n : \sigma_n$

$$\mathcal{E}(R(X_1, \ldots, X_n)) \Rightarrow_T \lambda x_1 \ldots \lambda x_n \mathcal{E}(r(x_1, \ldots, x_n))(X_1, \ldots, X_n) \text{ where } \{r := R\} \tag{10}$$

The term $\lambda x_1 \ldots \lambda x_n \mathcal{E}(r(x_1, \ldots, x_n))$ represents the characteristic function of the relation denoted by r, and thus, by R. While

$$\mathcal{E}(R(X_1, \ldots, X_n)) \Rightarrow \mathcal{E}(r) \text{ where } \{r := R(X_1, \ldots, X_n)\} \tag{11}$$

$$\mathcal{E}(like(j)(m)) \Rightarrow_T \lambda x_1 \lambda x_2 \mathcal{E}(r(x_1, x_2))(j, m) \text{ where } \{r := like\} \tag{12}$$

$$\mathcal{E}(believe(j)(q)) \Rightarrow \mathcal{E}(r) \text{ where } \{r := believe(j, q)\} \tag{13a}$$

$$\mathcal{E}(believe(j)(q)) \Rightarrow_T \lambda x_1 \lambda x_2 \mathcal{E}(r(x_1, x_2))(j, q) \text{ where } \{r := believe\} \tag{13b}$$

Definition

For any attitude constant B and any assignment system A_0 where $\{p_1 := A_1, \ldots, p_i := A_i, \ldots, p_j := A_j, \ldots, p_n := A_n\}$,

1. if $B(\vec{u}, p_j, \vec{v})$ occurs in some A_i, then p_j is in the scope of B
2. if p_k is in the scope of B, and p_r occurs in A_k, then p_r is in the scope of B.

Restricted Compositionality of $\mathcal{E}$

Let $(A_0 \text{ where } \{p_1 := A_1, \dots, p_n := A_n\}) : \widetilde{p}$ be such that

1. it is irreducible, and
2. $A_{i_1}, \dots, A_{i_k}$ are all the terms, such that $\mathcal{E}(A_j) \Rightarrow_T B_j$ and p_j is not in the scope of any attitude constant $(j \in \{0, \dots, n\})$.

Then, $\mathcal{E}(A_0 \text{ where } \{p_1 := A_1, \dots, p_n := A_n\}) \Rightarrow_T \mathrm{cf}(E)$, where

1. if A_0 is a proper term

$$E \equiv (\mathcal{E}(p_0) \text{ where } \{p_0 := A_0, p_1 := A_1 \dots, p_n := A_n\})\{A_{i_1} :\equiv B_{i_1}, \dots, A_{i_k} :\equiv B_{i_k}\} \tag{14a}$$

2. if A_0 is immediate (i.e., not a proper term)

$$E \equiv (\mathcal{E}(A_0) \text{ where } \{p_1 := A_1 \dots, p_n := A_n\})\{A_{i_1} :\equiv B_{i_1}, \dots, \dots, A_{i_k} :\equiv B_{i_k}\} \tag{14b}$$

$$\text{John believes that Mary is happy.} \xrightarrow{\text{render}} \tag{15a}$$

$$A \equiv believe(john)(happy(mary)) \ : \ \widetilde{\mathrm{p}} \tag{15b}$$

$$\Rightarrow_{cf} \ believe(j)(q) \, \text{where} \, \{m := mary, \, j := john, \\ q := happy(m)\} \tag{15c}$$

$$\mathcal{E}(believe(john)(happy(mary))) \ : \ \widetilde{\mathrm{t}} \tag{16a}$$

$$\Rightarrow \mathcal{E}(P) \, \text{where} \, \{P := believe(john)(happy(mary))\} \tag{16b}$$

$$\Rightarrow_{cf} \ \mathcal{E}(P) \, \text{where} \, \{P := believe(j)(q), \, m := mary, \, j := john, \\ q := happy(m)\} \tag{16c}$$

$$\text{cf}(A) \Rightarrow_T \mathcal{E}(P) \, \text{where} \, \{P := \lambda x_1 \lambda x_2 \mathcal{E}(r(x_1, x_2))(j, q), \\ m := mary, \, j := john, \\ r := believe, q := happy(m)\} \tag{17}$$

The Big Picture in NLP: Simplified and Approximated, but Realistic

- Semantics of NL: via "logical forms"

$$\underbrace{\text{Syn of NL} \Longleftrightarrow \text{Syn of } L^{\lambda}_{ar}/L^{\lambda}_{r} \Longrightarrow \text{Canonical Terms} \Longrightarrow \text{Denotations}}_{\text{SynSem}}$$

- Translation

$$\text{Lexicon of NL}_1 \iff \text{Syn of NL}_1 \xrightarrow{\text{render}} L^{\lambda}_{ar}/L^{\lambda}_{r} \text{ Terms}$$

$$\Downarrow \text{ Reduction}$$

$$L^{\lambda}_{ar}/L^{\lambda}_{r} \text{ Canonical Terms}$$

$$\downarrow \text{ (possible modifications)}$$

$$\text{Lexicon of NL}_2 \iff \text{Syn of NL}_2 \xleftarrow{\text{render}^{-1}} L^{\lambda}_{ar}\text{Canonical Terms}$$

The Start