SOME PROPERTIES OF COMPUTABLE NUMBERINGS IN VARIOUS LEVELS OF DIFFERENCE HIERARCHY

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n-computable enumerable sets

Definition

We call a set $A \subseteq \omega$ is n-computable enumerable if there are uniformly computable sequence of sets $\{A_s\}_{s \in \omega}$ such for all x,

$$x \notin A_0$$
$$A(x) = \lim_s A_s(x)$$
$$|\{s \in \omega | A_{s+1} \neq A_s\}| \leq n$$

Ershov's hierarchy

Let S – univalent notation system for constructive ordinals, $A \subseteq \omega$ and α – ordinal, which has notation a in S.

Definition

Set $A \subseteq \omega$ in level Σ_α^{-1} of Ershov's hierarchy (or A is Σ_α^{-1}-set), if there exist partially computable function Ψ, and for all x,

$$x \in A \rightarrow \exists\lambda(\Psi(\lambda, x)\downarrow) \text{ and } A(x) = \Psi((\mu\lambda_{<\alpha})_S(\Psi((\lambda)_S, x)\downarrow, x))$$

$$x \notin A \rightarrow \text{ or } \forall\lambda(\Psi(\lambda, x)\uparrow), \text{ or } \exists\lambda(\Psi(\lambda, x)\downarrow) \text{ and } A(x) = \Psi((\mu\lambda_{<\alpha})_S(\Psi((\lambda)_S, x)\downarrow, x)).$$

Ershov's hierarchy

Definition

A in level Π_α^{-1} of Ershov's hierarchy, if $\overline{A} \in \Sigma_\alpha^{-1}$

A in level Δ_α^{-1} of Ershov's hierarchy, if A and $\overline{A}$ are Σ_α^{-1}-sets, in other words $\Delta_\alpha^{-1} = \Sigma_\alpha^{-1} \bigcap \Pi_\alpha^{-1}$.

Numbering

Definition

Numbering of family S is a map ν from ω onto the family S

Definition

Numbering η is called Σ_{α}^{-1}-computable, if set $\{<x, y>|y \in \eta x\}$ is a Σ_{α}^{-1}-set and Δ_{α}^{-1}-computable, if $\{<x, y>|y \in \eta x\}$ in level Δ_{α}^{-1}.

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Numberings

Definition

Numbering η is called Friedberg numbering, if for all $n \neq m$ $\eta_n \neq \eta_m$.

Definition

Numbering μ is called minimal, if for all numberings ν_n from reducing ν to μ goes, that ν is equivalent to μ.

Preposition

There is no universal computable function for family of all computable sets.

Theorem

There is no Δ_α^{-1}-computable numbering for family of all Δ_α^{-1}-sets.

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Friedberg theorem

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There is effective enumeration of the family of all computable enumerable sets without repetition.

Theorem

(Goncharov, Lemp, Solomon) For all n there is Σ_n^{-1}-computable Friedberg numbering for family of all Σ_n^{-1}-sets.

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Friedberg numberings

Theorem

For all n there is Σ_{2n}^{-1}-computable Friedberg numbering for family of all Σ_n^{-1}-sets. And there is computable function, which m-reduces Friedberg numbering for family of all Σ_{n-1}^{-1}-sets to Friedberg numbering for family of all Σ_n^{-1}-sets. Moreover, numbering and function are constructed uniformly on n.

Union of all finite levels of difference hierarchy

Theorem

Let β^n is numbering, which is constructed in previous theorem. Define γ:

$$\gamma_n = \beta^{n_1}_{n_2},$$

where $n =< n_1, n_2 >$.
γ_n is Δ_ω^{-1}-computable minimal numberings for family of all sets from $\bigcup\limits_{k\in\omega} \Sigma_k^{-1}$.

Thanks for your attention!:)