

- ▶ STEPHAN KREUTZER, *Algorithmic meta-theorems: upper and lower bounds*.  
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In 1990, Courcelle proved a fundamental theorem stating that every property of graphs definable in monadic second-order logic can be decided in linear time on any class of graphs of bounded tree-width. This theorem is the first of what is today known as algorithmic meta-theorems, that is, results of the form: every property definable in a logic  $L$  can be decided efficiently on any class of structures with property  $P$ .

Such theorems are of interest both from a logical point of view, as results on the complexity of the evaluation problem for logics such as first-order or monadic second-order logic, and from an algorithmic point of view, where they provide simple ways of proving that a problem can be solved efficiently on certain classes of structures.

Following Courcelle's theorem, several meta-theorems have been established, primarily for first-order logic with respect to properties of structures derived from graph theory. (See [1, 2] for recent surveys.) In this talk I will present recent developments in the field and illustrate the key techniques from logic and graph theory used in their proofs.

So far, work on algorithmic meta-theorems has mostly focused on obtaining tractability results for as general classes of structures as possible. The question of finding matching lower bounds, that is, intractability results for monadic second-order or first-order logic with respect to certain graph properties, has so far received much less attention. Tight matching bounds, for instance for Courcelle's theorem, would be very interesting as they would give a clean and exact characterisation of tractability for MSO model-checking with respect to structural properties of the models. In the second part of the talk I will present a recent result in this direction showing that Courcelle's theorem can not be extended much further to classes of unbounded tree-width.

[1] M. Grohe. *Logic, graphs, and algorithms*. In T. Wilke J. Flum, E. Grädel, editor, *Logic and Automata – History and Perspectives*. Amsterdam University Press, 2007.

[2] S. Kreutzer. *Algorithmic meta-theorems*. to appear. Preprint available at CoRR abs/0902.3616, <http://arxiv.org/abs/0902.3616>, 2008.